EE123 Spring 2016 Miki Lustig
Digital Signal Processing

**Assignment 3**

Due February $12^{th}$ 2016

1. Read Chapters 8,9.1-9.5 Oppenheim and Schafer, 3rd ed.
2. Problem 8.24, Oppenheim and Schafer, 3rd ed.
3. Problem 8.67 Oppenheim and Schafer, 3rd ed. (Not easy!)
4. Problem 8.69 Oppenheim and Schafer, 3rd ed.
5. *Symmetries of the DFT*

   Let $f[n]$ be an 8-sample sequence with the $DFT\{f[n])\} = F[k]$. We know some of the values of $f[n]$:

   $$f[n] = \{1 \;\; i \;\; -1 \;\; -i \;\; 1 \;\; ? \;\; ? \;\; ?\}$$

   Other values, indicated by "?", are unknown. We want to find these unknown values based on the what we know about the symmetry of the signal or the symmetry of its DFT.

   Determine the rest of $f[n]$ for each of the following cases. If $F[k]$ cannot possess the given property, explain.

   (a) $F[k]$ is even (i.e. $F[k] = F[((-k))_8]$).
   (b) $F[k]$ is odd (i.e. $F[k] = -F[((-k))_8]$).
   (c) $F[k]$ is imaginary-valued.
   (d) $F[k]$ is real-valued.

6. *Faster DFT's?*

   Let $f[n]$ and $g[n]$ be N-point real-valued sequences with DFT's of $F[k]$ and $G[k]$ respectively. Consider the possibility of computing both of their DFT's simultaneously by using an N-point complex DFT.

   One idea is to construct $h[n] = f[n] + jg[n]$. If $DFT\{h[n]\} = H[k]$, find separate expressions (if possible) for $F[k]$ and $G[k]$ in terms of $H[k]$. If it is not possible to separate $F[k]$ and $G[k]$ from $H[k]$, explain why.

7. *Diagonalizing circulant matrices* A circulant matrix is a matrix of the form

   $$H_m = \begin{bmatrix} c_1 & c_2 & \dots & c_{n-1} & c_n \\ c_n & c_1 & \dots & c_{n-2} & c_{n-1} \\ & & \dots & & \\ c_2 & c_3 & \dots & c_n & c_1 \end{bmatrix},$$

   i.e. each row is a circular shift of the row above it. Show that the DFT matrix diagonalizes all circulant matrices. (Hint: recall the shift and modulation properties of the DFT)

8. Problem 9.47, parts a,b,c,d Oppenheim and Schafer, 3rd ed.