OCT 3, 03

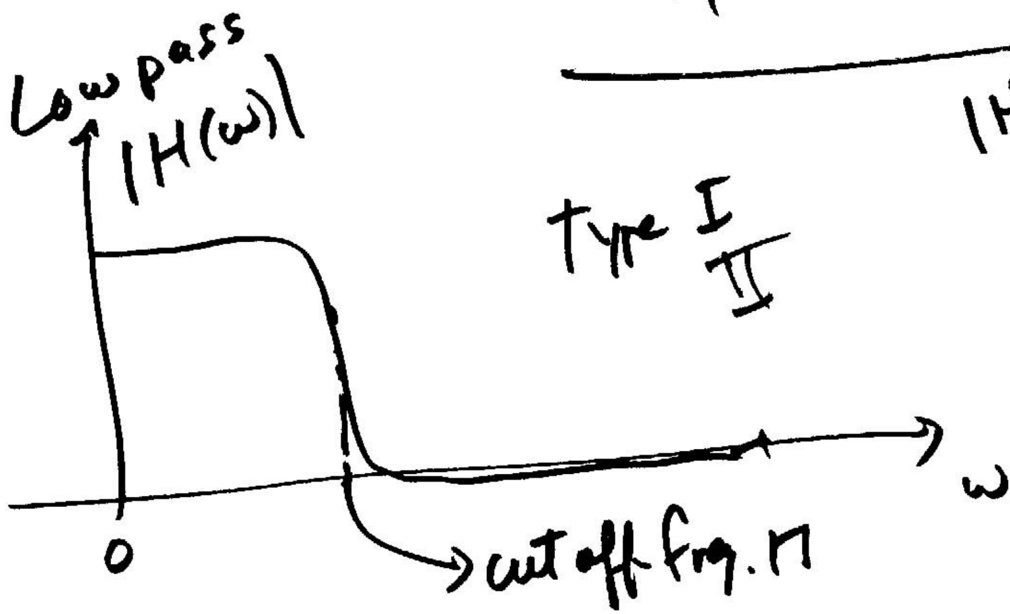
Generalized Linear Phase Filter

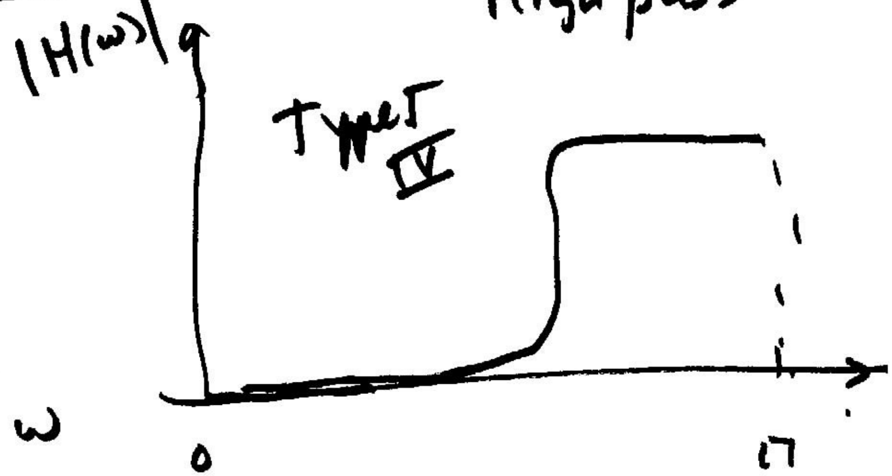
$$H(\omega) = \underbrace{H_m(\omega)}_{\text{Real.}} \, e^{-j(\beta - \alpha\omega)}$$

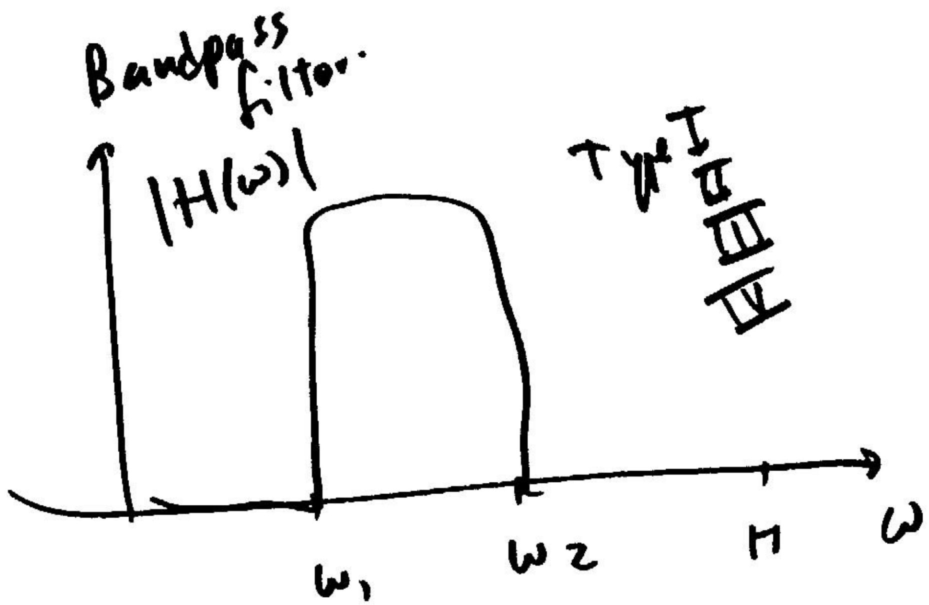
O & S $\longrightarrow$ 5.7

	Symmetry	N # Taps	α	β	$H_m(\omega)$	Constraint. $H_m(\omega)e^{j\beta}$
Type I	$h(n) = h(N-1-n)$	odd	$\frac{N-1}{2}$	0	$\sum_{0}^{\frac{N-1}{2}} a(n) \cos \omega n$	Real.
Type II	$h(n) = h(N-1-n)$	even	$\frac{N-1}{2}$	0	$\sum_{n=1}^{N/2} b(n) \cos \omega(n-\frac{1}{2})$	Real. $H(\pi) = 0$
Type III	$h(n) = -h(N-1-n)$	odd	$\frac{N-1}{2}$	$\pi/2$	$\sum_{n=1}^{\frac{N-1}{2}} c(n) \sin \omega n$	purely imaginary $H(0) = 0$ $H(\pi) = 0$
Type IV	$h(n) = -h(N-1-n)$	even.	$\frac{N-1}{2}$	$\pi/2$	$\sum_{n=1}^{N/2} d(n) \sin \omega(n-\frac{1}{2})$	purely Imaginary. $H(0) = 0$

4 Classes of Filters







	Low Pass	High Pass.	Band pass.	BandsTop.
I	✓	✓	✓	✓
II	✓	✗	✓	✗
III	✗	✗	✓	✗
IV	✗	✓	✓	✗


|H(ω)|
0
ω

Quantization

- In implementing things finite precision.

  1. A/D Conversion. finite # of bits of signal per sample.
  2. coeff. quantization in filter implementation.
  3. ~~##~~ flowgraph : arithmetic

- Need a representation scheme for finite precision.

- Real # is in Two's complement

∞ precision

finite precision.

$$x = X_m \left(-b_0 + \sum_{i=1}^{\infty} b_i \, 2^{-i} \right)$$

real #

scale factor

$|x| < X_m$

sign bit

0 or 1

$b_0 = 0 \Rightarrow x$ pos.

$b_0 = 1 \Rightarrow x$ is neg.

0 or 1

- Want finite # of bits $(B+1)$: We get

$$\hat{x} = Q_B[x] = X_m \underbrace{\left(-b_0 + \sum_{i=1}^{B} b_i \, 2^{-i} \right)}_{\hat{x}_B}$$

quantized version of x

$$\hat{x} = X_m \hat{x}_B$$

$\hat{x}$ = quantized version of x.

$\Rightarrow$ smallest difference between #s is $2^{-B} X_m = \Delta$

$$|\hat{x}| < X_m$$

$$\hat{x}_B = b_0 \,\square\, b_1 b_2 \ldots \; b_B$$

binary point

Start with a real #
can quantize it either by rounding or by Truncation

rounding + Truncation: nonlinear.
memoryless operation.

Define. $e = Q_B[x] - x$

Two's complement: rounding $-\Delta/2 < e < \Delta/2$

Truncation $\Delta \le e < 0$

overflow: 4 bits. 3 bits + sign bit.

sign bit

$$\begin{array}{lcl} 0111 & = & 7 \\ 0001 & = & 1 \\ \hline 1000 & \longrightarrow & -8 \end{array}$$

negative

- Trade off between quantization error, dynamic range, for a given # of bits B.

= To minimize overflow, (with fixed # of bits B), we must increase $X_m \Longrightarrow \Delta \uparrow \longrightarrow e \uparrow \Longrightarrow$ quantization error goes up.

$\Longrightarrow$ Trade-off dynamic range $\longleftrightarrow$ quantization error (with fixed # bits).

two's complement : if sum of N #s doesn't overflow, Then result is correct even if intermediate results overflow.

Quantization in Implementing System

Fig 6.39

effects of coefficient quantization:

$$H(z) = \frac{\sum b_k z^{-k}}{\sum a_k z^{-k}}$$

$$\hat{H}(z) = \frac{\sum \hat{b}_k z^{-k}}{\sum \hat{a}_k z^{-k}} \begin{matrix} \rightarrow \text{zeros} \\ = \text{poles} \end{matrix}$$

Direct form I

II

Cascade

Parallel

( )