8/29/03 EE123

LTI Systems

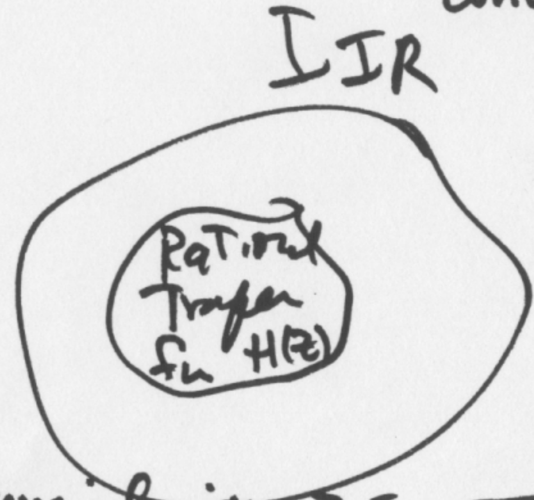
impulse response entirely & uniquely characterizes

$\delta(n) \rightarrow \boxed{LTI} \rightarrow h(n) \leftarrow$ impulse response.

$x(n) \rightarrow \boxed{LTI} \rightarrow y(n) = \sum_k h(k)\, x(n-k)$

convolution

$h(n) \rightarrow$ FIR, IIR



H(z) Rational Transfer fn

$\Rightarrow H(z) =$ Ratio of 2 polynomials in $z = \frac{P(z)}{Q(z)} =$

$H(z) = \frac{Y(z)}{X(z)} = \frac{P(z)}{Q(z)} \Rightarrow$ Difference eqn. $\Rightarrow$

LTI

Causiety:

LTI is causal. $\longleftrightarrow$ $h(n)=0 \quad n<0$

Stability

BIBO = Bounded input
Bounded ~~input~~ output

BIBO stable LTI $\longleftrightarrow$ $\sum_{n=-\infty}^{+\infty} |h(n)| < \infty$

Eigenfunctions

Eigenvectors: $A\vec{x} = \alpha\vec{x}$

scaling factor = eigenvalue.

Eigenfunction

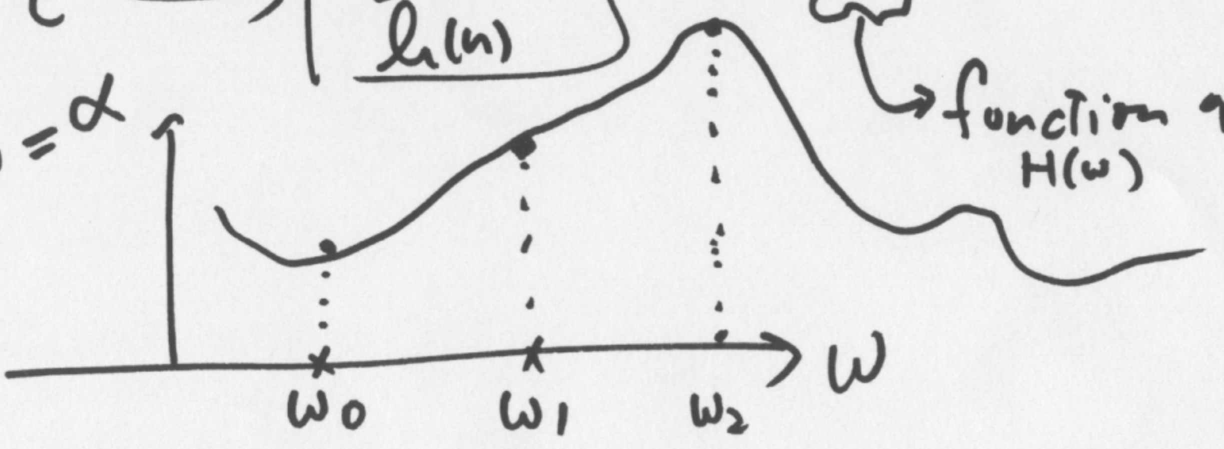
$$\Phi(n) \longrightarrow \boxed{\text{system}} \longrightarrow \alpha\,\Phi(n)$$

LTI systems: eigenfunction are $e^{j\omega n}$

$$e^{j\omega n} \longrightarrow \boxed{\begin{matrix}\text{LTI}\\ h(n)\end{matrix}} \longrightarrow \underbrace{\alpha}_{\text{function of } \omega,\ H(\omega)} e^{j\omega n}$$

$H(\omega) = \alpha$

freq. Response.



Can show: Using convolution

$$x(n) = e^{j\omega n} \longrightarrow \boxed{\text{LTI } h(n)} \longrightarrow y(n)$$

$$y(n) = \sum_k e^{j\omega k} h(n-k) =$$

$$\boxed{H(\omega) = \sum_n h(n) e^{-j\omega n}}$$

D.T.F.T. of $h(n)$.

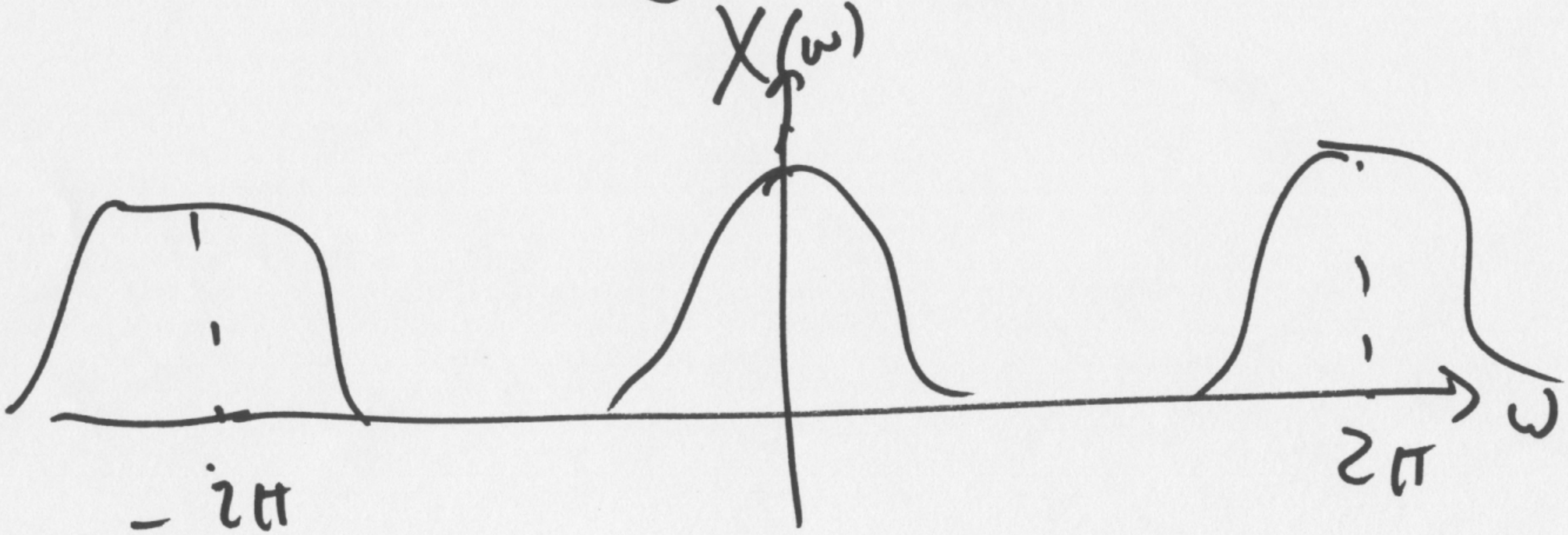
$x(n)$: $\text{D.T.F.T.}\{x(n)\} = X(\omega) = \sum_n x(n) e^{-j\omega n}$

D.T.F.T.

n = integer
ω = continuous

$$X(\underline{\omega}) = \sum x(n)\, e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(\omega)\, e^{j\omega n}\, d\omega.$$



$$x \longrightarrow \boxed{\begin{matrix} h \\ \text{LTI} \end{matrix}} \longrightarrow y$$

$$\left.\begin{aligned} x * h &= y \\ Y(\omega) &= X(\omega)\, H(\omega) \end{aligned}\right\}$$

Parsaveli's Thm:

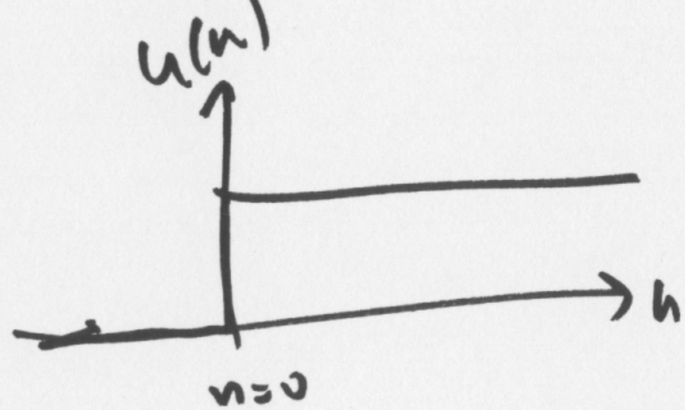
$$\sum_n x(n)\, y^*(n) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(\omega)\, Y(\omega)$$

special case $x(n) = y(n)$

$$\sum_n |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} |X(\omega)|^2 \, d\omega$$

Ex: $\delta(n) \longrightarrow 1$

$\left(\frac{1}{2}\right)^n u(n) \longrightarrow \dfrac{1}{1-\frac{1}{2}e^{-j\omega}}$



Geometric series.

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega n} =$$

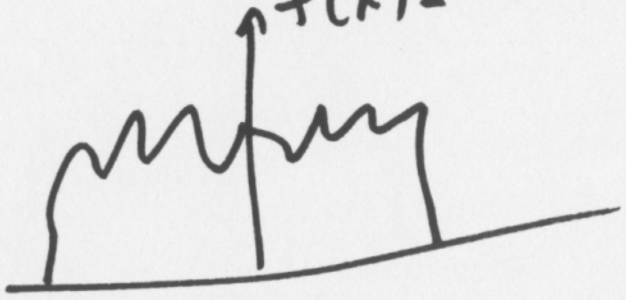
$$1 + x + x^2 + \dots = \frac{1}{1-x} \quad \text{if } |x|<1$$

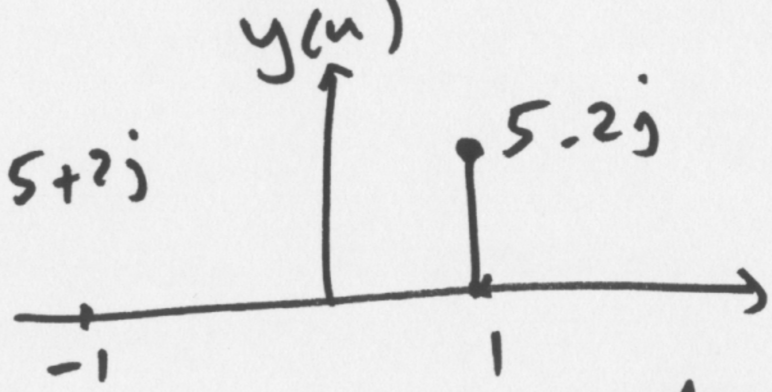
$2^n u(n) \longrightarrow$ Doesn't exist.

Symetry Props

- conjugate symetric : $Y(n) = Y^*(-n)$

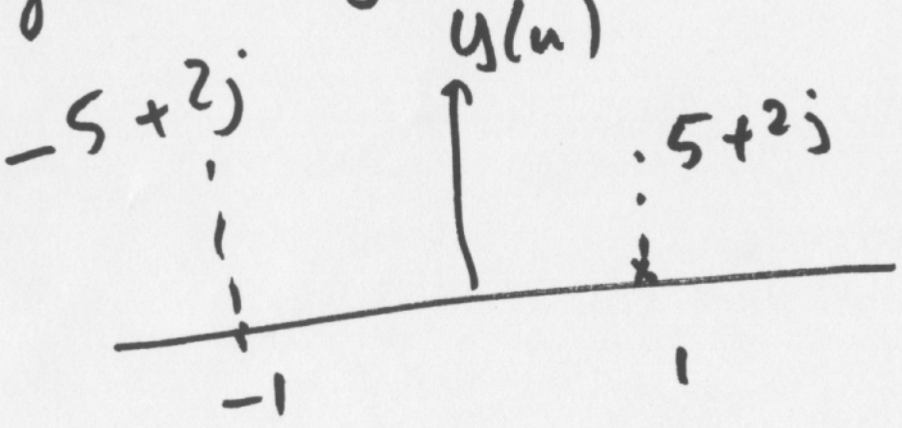
$f(x) = f(-x)$





$y(n) = -y^*(-n)$

- Conjugate antisymetric



Any $x(n)$ can be decomposed into sum of conjugate symmetric + conjugate antisymmetric.

$X_e(n) \triangleq$ conjugate symmetric of $x(n)$

$$= \frac{1}{2}\left[x(n) + x^*(-n) \right]$$

$X_o(n) \triangleq$ conjugate antisymmetric. $x(n)$

$$= \frac{1}{2}\left[x(n) - x^*(-n) \right]$$

$$x(n) = X_e(n) + X_o(n)$$

$$X(\omega) = X_e(\omega) + X_o(\omega)$$

$$X_e(\omega) \triangleq \frac{1}{2}\left[X(\omega) + X^*(-\omega)\right]$$

$$X_o(\omega) \triangleq \frac{1}{2}\left[X(\omega) - X^*(-\omega)\right]$$

Symmetry Properties

$$x(n) \xleftrightarrow{\text{DTFT}} X(\omega)$$

$$\mathrm{Re}\{x(n)\} \xleftrightarrow{\text{D.T.F.T.}} X_e(\omega)$$

$$x_e(n) \xleftrightarrow{\text{D.T.F.T.}} \mathrm{Re}\{X(\omega)\}$$

$$j\,\mathrm{Im}[x(n)] \xleftrightarrow{\text{D.T.F.T.}} X_o(\omega)$$

$$x_o(n) \xleftrightarrow{\text{D.T.F.T.}} j\,\mathrm{Im}\{X(\omega)\}$$

if $x(n)$ is real $\Rightarrow$ $x(n) = \mathrm{Re}\{x(n)\}$

$$X(\omega) = X_e(\omega) = \frac{1}{2}\left[X(\omega) + X^*(-\omega)\right]$$

$$\Longrightarrow X(\omega) = X^*(-\omega)$$

"

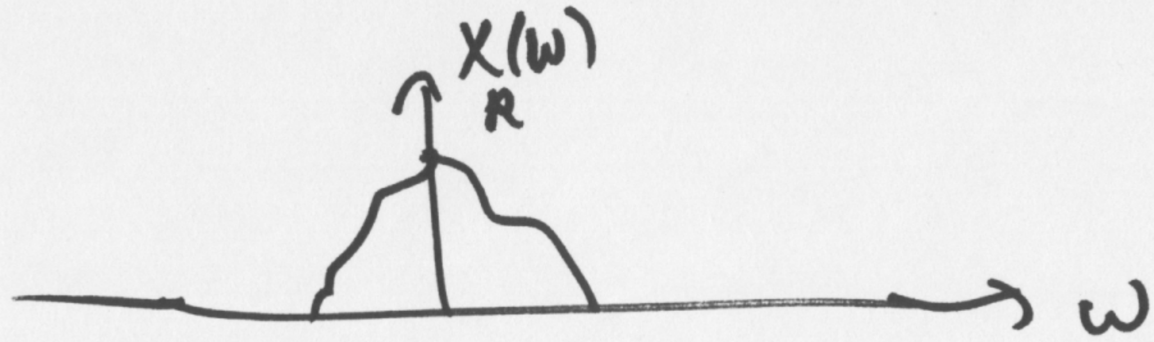
If seq. is real $\Longrightarrow$ F.T is conjugate symmetric

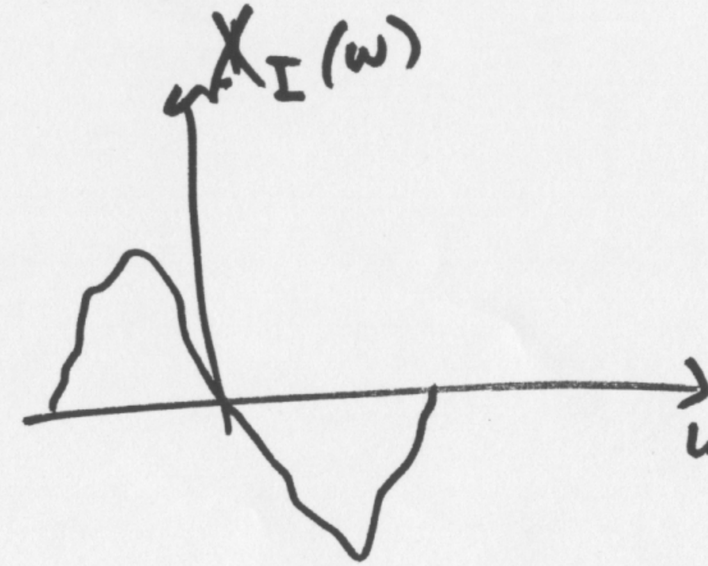
$$X(\omega) = X^*(-\omega) \Longrightarrow$$

$$X_R(\omega) = Re\{X(\omega)\} = Re\{X^*(-\omega)\}$$

$$\Longrightarrow (1)\ X_R(\omega) = X_R(-\omega)$$

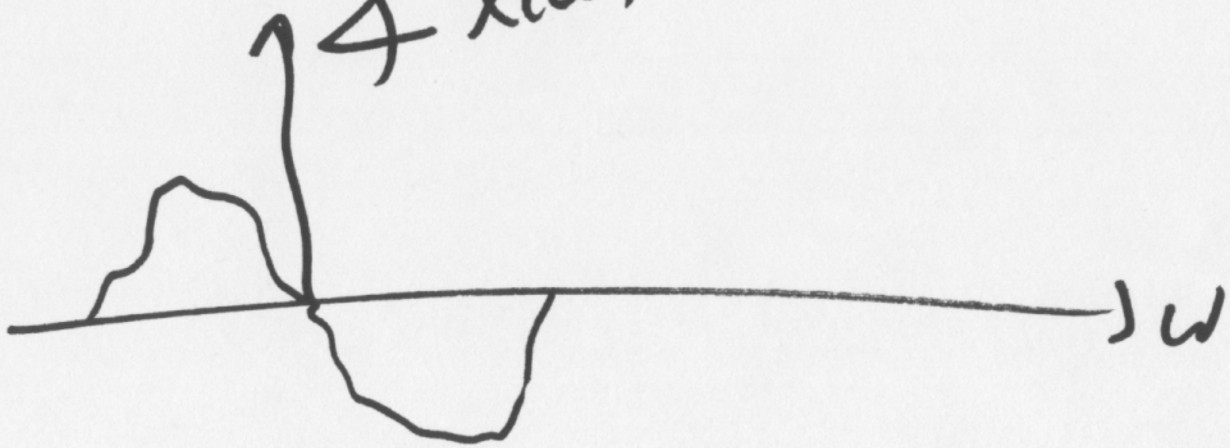
$$Im\{X(\omega)\} = X_I(\omega) \Longrightarrow (2)\ X_I(\omega) = -X_I(-\omega)$$





(3) $|X(\omega)| = |X(-\omega)| \Longrightarrow$

(4) $\measuredangle X(\omega) = -\measuredangle X(-\omega)$



Convergence Issues

$$X(\omega) = \text{D.T.F.T.}\{x(n)\}$$
$$= \sum_{n=-\infty}^{+\infty} x(n)\, e^{-j\omega n}$$

Q: Does it converge?

if yes what does it converge To

$\longrightarrow Y(\omega)$

Q: If it converged $Y(\omega)$, plug $Y(\omega)$

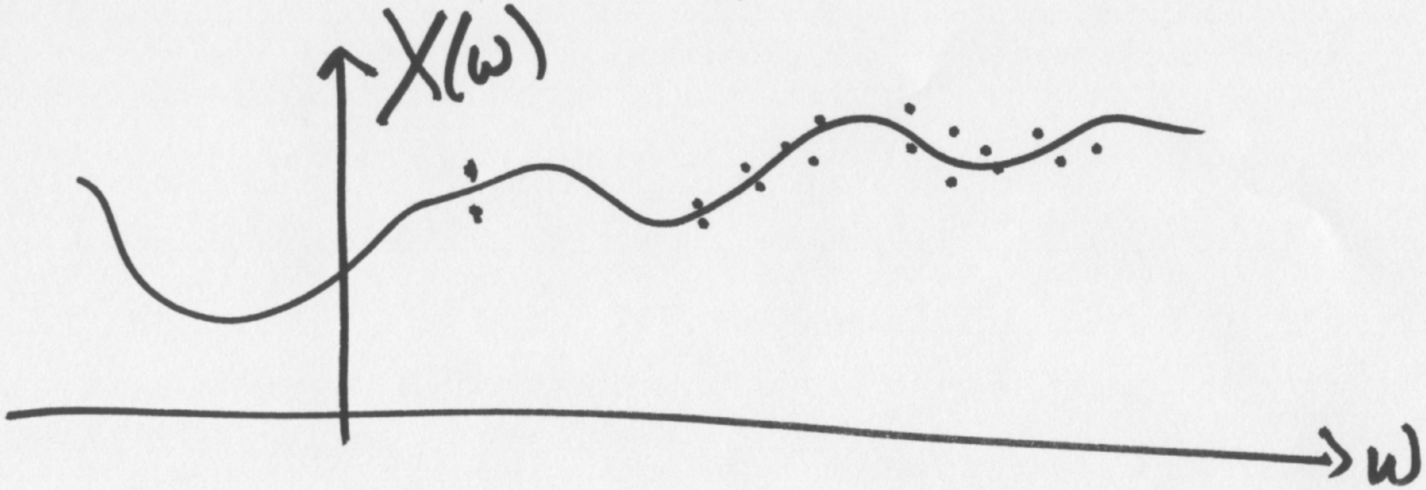
$$\int Y(\omega)\, e^{j\omega n}\, d\omega \overset{??}{=} x(n)$$

Def uniform Convergence: $\sum_n x(n) e^{-j\omega n}$

converges uniformly if $\exists$ a continous function of ω, called $X(\omega)$ such That.

$\forall\ \epsilon, \omega_{\epsilon>0}$, $\exists\ n_0$ s.That.

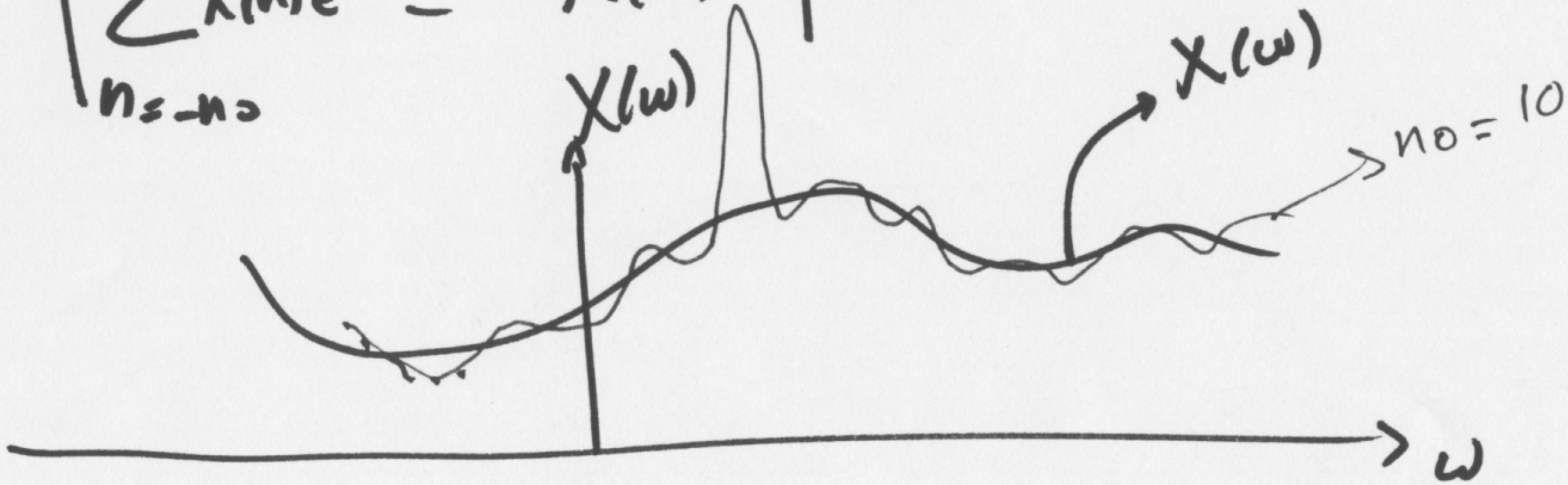
$$\left| \sum_{n=-n_0}^{+n_0} x(n) e^{-j\omega n} - X(\omega) \right| < \epsilon$$



(mean square error) convergence:

$\sum_n x(n) e^{-j\omega n}$ converges in mse sense to $X(\omega)$ if $\forall \epsilon$, $\exists n_0$ s.th.

$$\int_{-\pi}^{\pi} \left| \sum_{n=-n_0}^{+n_0} x(n) e^{-j\omega n} - X(\omega) \right|^2 d\omega < \epsilon$$



1) If seq $x(n)$ is abs. summable.

$\sum_n |x(n)| < \infty$ then $\sum_n x(n) e^{-j\omega n}$

converges uniformly. DT.F.T. exists.

DTFT is continuous function. whatever it converges to $X(\omega)$ is D.T.F.T of $x(\omega)$

$\Rightarrow$ plug into $\int X(\omega) e^{j\omega n} d\omega \rightarrow x(n)$

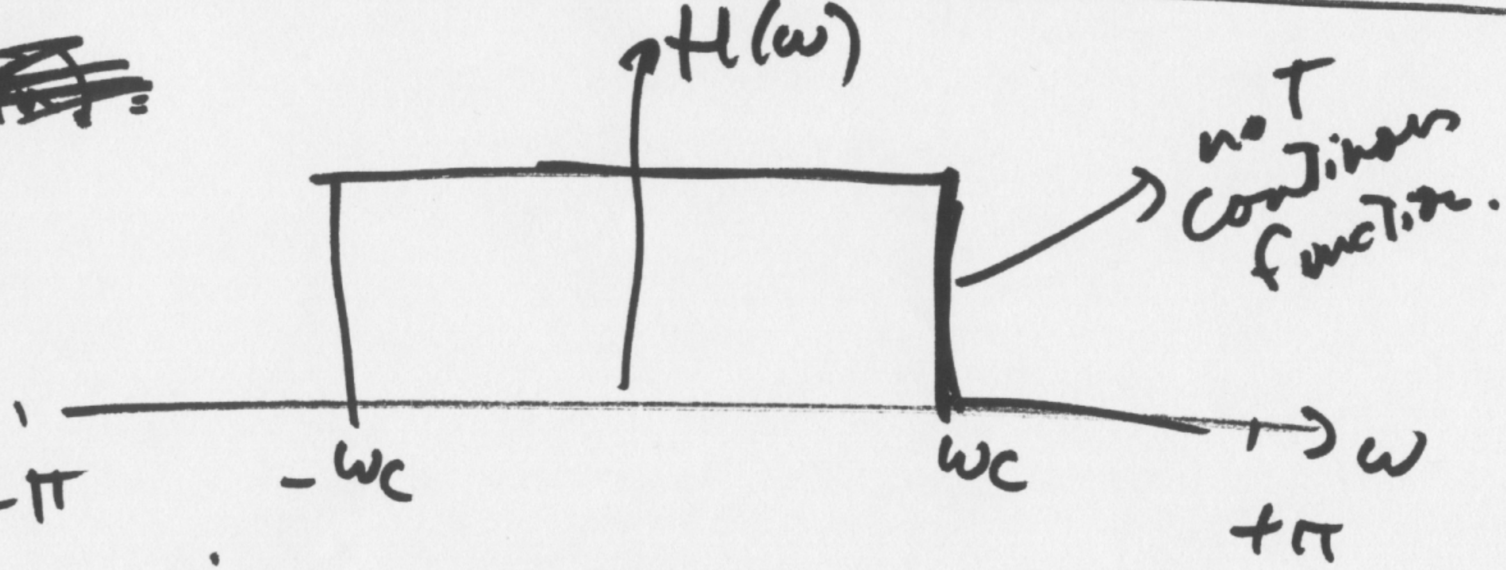
Ex $\left(\frac{1}{2}\right)^n u(n) = x(n) \longrightarrow \frac{1}{1 - \frac{1}{2} e^{-j\omega}}$

$2^n u(n) = x(n) \longrightarrow$ F.T. doesn't exist

(2) If seq $x(n)$ is square summable.

$\sum_n x^2(n) < \infty \Longrightarrow$ converge in mse sense.

Ex ~~X~~



$\int_{-\pi}^{\pi} H(\omega) e^{j\omega n} \, d\omega = \frac{\sin \omega_c n}{\pi n} = h(n)$

$\sum_n \frac{1}{n} \rightarrow$ diverge $\Rightarrow h(n)$ not abs. summable.

$\Longrightarrow$

Is $h(n)$ square summable $\Rightarrow$ yes.

$\sum \frac{1}{n^2}$ converges.

$\Rightarrow \sum h(n) e^{-j\omega n}$ converges in mse sense

Can show. $\sum_{n=-n_0}^{n_0} h(n) e^{-j\omega n} = \frac{1}{2}$

for all n_0

Show 0 vs Fig 2.20 1989 edition.