Sept. 10, 2003

Upsampling

- Increasing sampling rate by an integer factor

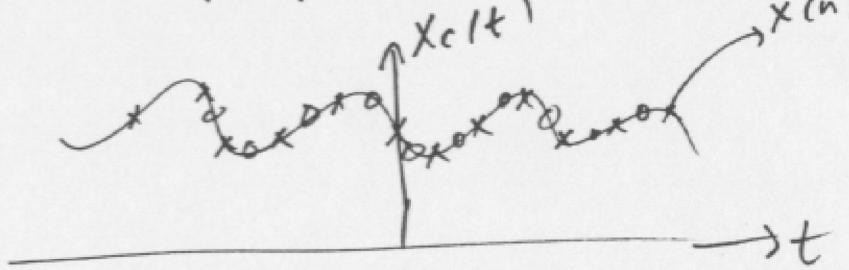
$$x_c(t) \longrightarrow \boxed{C/D} \longrightarrow x(n) = x_c(nT)$$

T

$$x_c(t) \longrightarrow \boxed{C/D} \longrightarrow x_i(n) = x_c\left(n\frac{T}{L}\right)$$

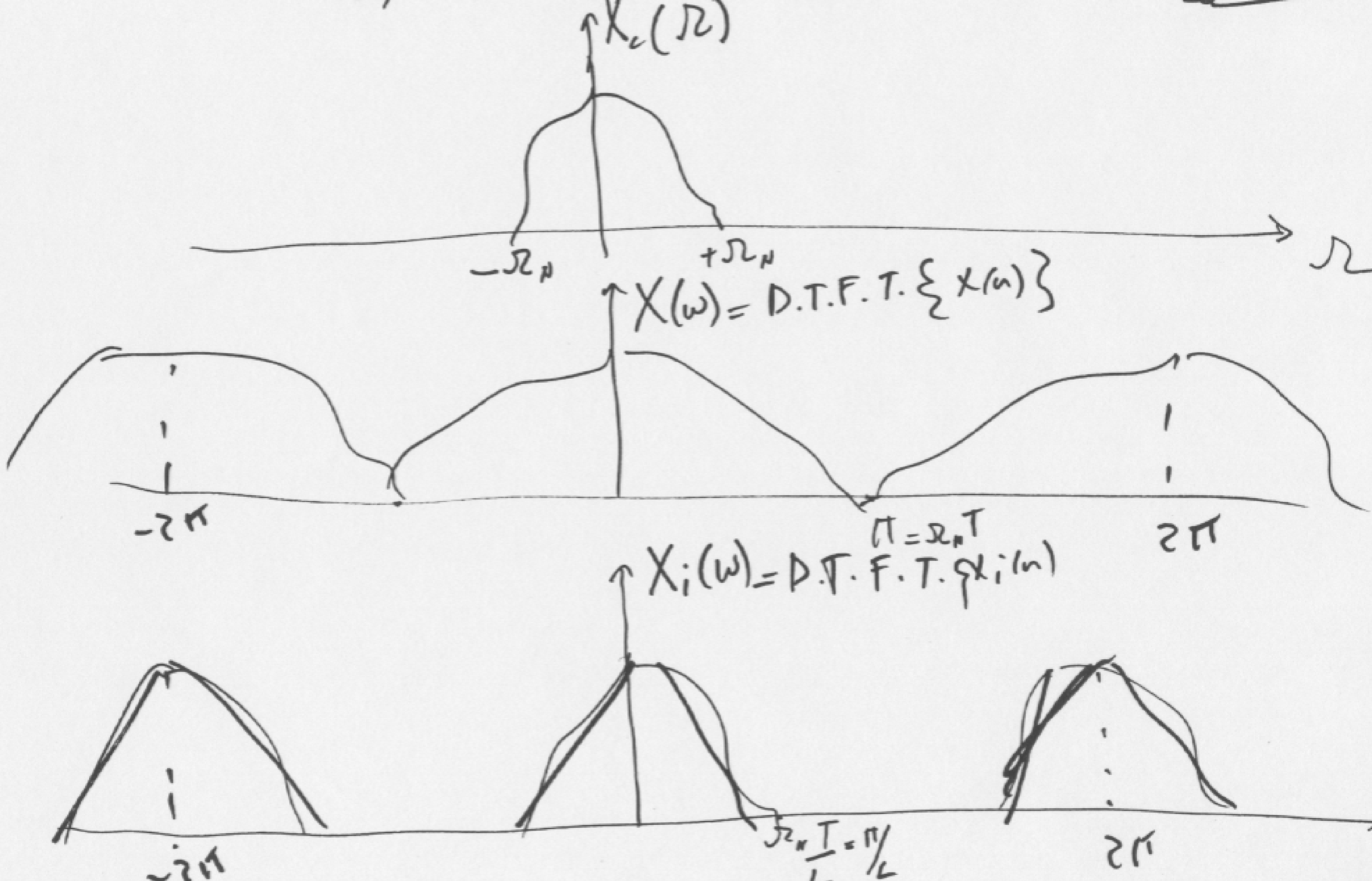
T/L

interpolated



Q: Can I obtain $x_i(n)$ from $x(n)$?

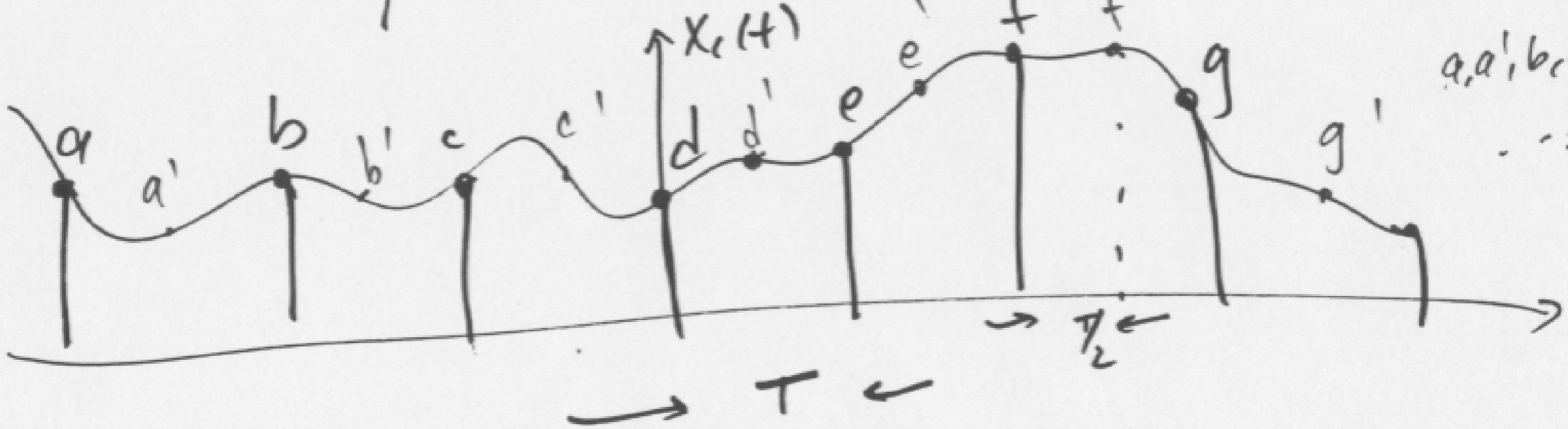
A: It depends. If $x_c(t)$ was sampled "fast" enough to obtain $x(n)$, <u>yes</u>.



Proposed Soln To go from $X(\omega)$ To $X_i(\omega)$

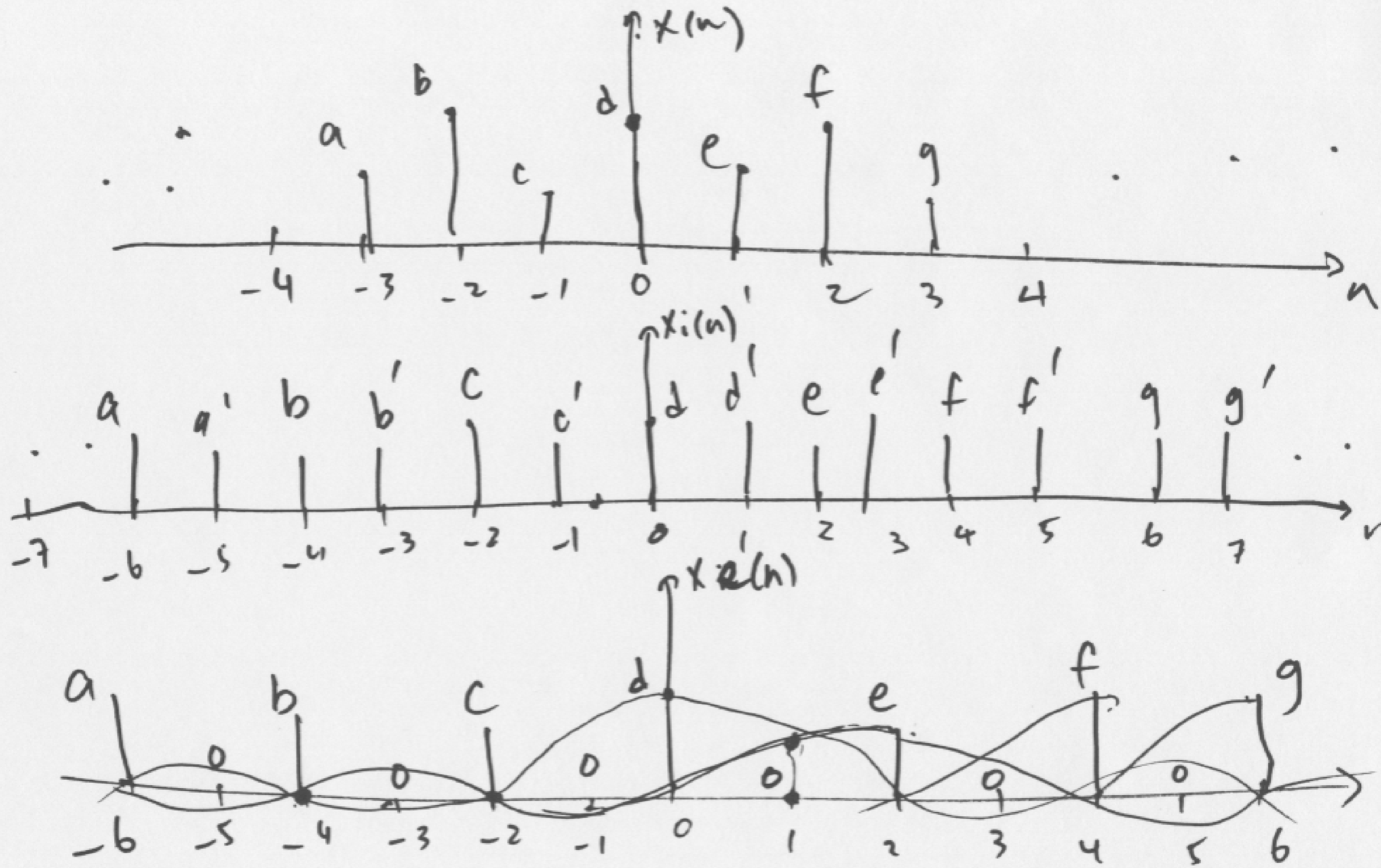
$$x(n) \rightarrow \boxed{\uparrow L} \xrightarrow{x_e(n)} \boxed{\begin{array}{c}\text{L.P.F.} \\ \text{gain } L \\ \text{cutoff } \pi/L\end{array}} \rightarrow x_i(n)$$

$$x_e(n) \triangleq \begin{cases} x\left(\frac{n}{L}\right) & n = 0, \pm L, \pm 2L, \ldots\ldots \\ 0 & \text{otherwise} \end{cases}$$



$a, b, d, c, e\ f\ g \rightarrow x(n)$

$a, a', b, b', c, c', \ldots \rightarrow x_i(n)$



Relate D.T.F.T. xe(n) to D.T.F.T. x(n).

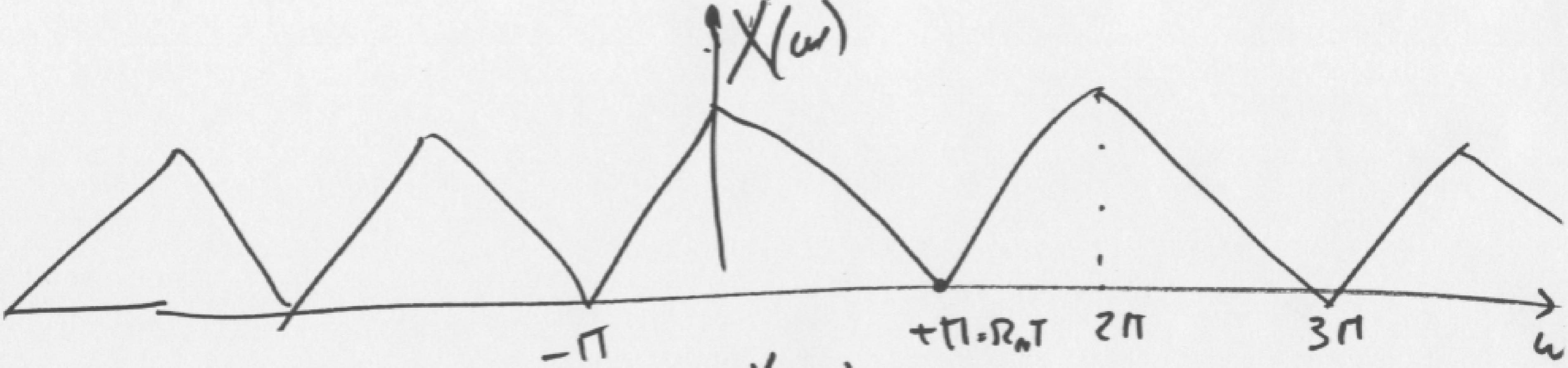
$$x_e(n) = \sum_{k=-\infty}^{+\infty} x(k)\, \delta(n - kL)$$

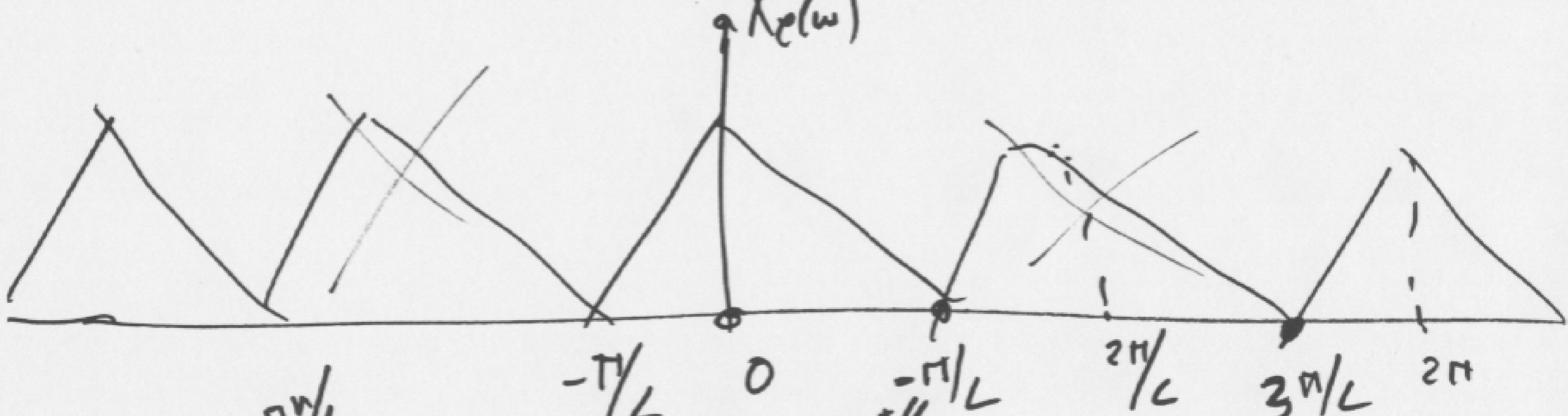
$$X_e(\omega) = \text{D.T.F.T}\{x_e(n)\} = \sum_{n=-\infty}^{+\infty} \left(\sum_{k=-\infty}^{+\infty} x(k)\, \delta(n-kL) \right) e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{+\infty} x(k) \underbrace{\sum_{n=-\infty}^{+\infty} \delta(n-kL)\, e^{-j\omega n}}_{e^{-j\omega L k}}$$

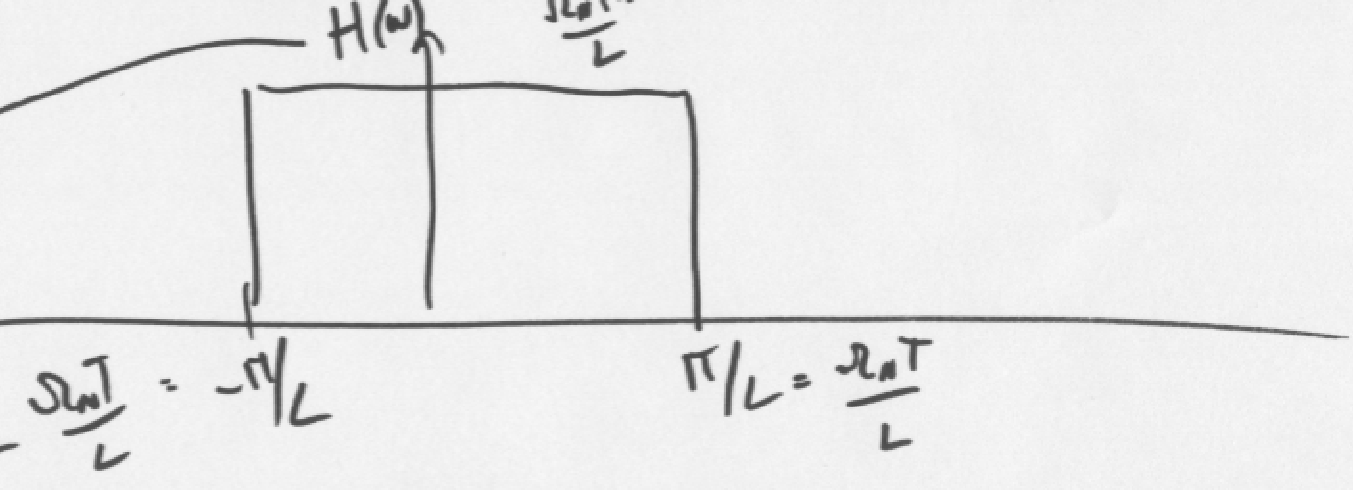
$$\left.\begin{aligned} X_e(\omega) &= \sum_{k=-\infty}^{+\infty} x(k)\, e^{-j\omega L k} \\ X(\omega) &= \sum_{k=-\infty}^{+\infty} x(k)\, e^{-j\omega k} \end{aligned}\right\} \Longrightarrow X_e(\omega) = \left[X(\omega) \right]_{\omega \to \omega L}$$

$$\boxed{X_e(\omega) = X(\omega L)}$$





$h(n) = \frac{\sin \pi n/L}{\pi n/L}$



Obtain $x_i(n)$ from $x_e(n)$ by just L.P.F. cutoff of π/L.

$$x_i(n) = x_e(n) * h(n)$$

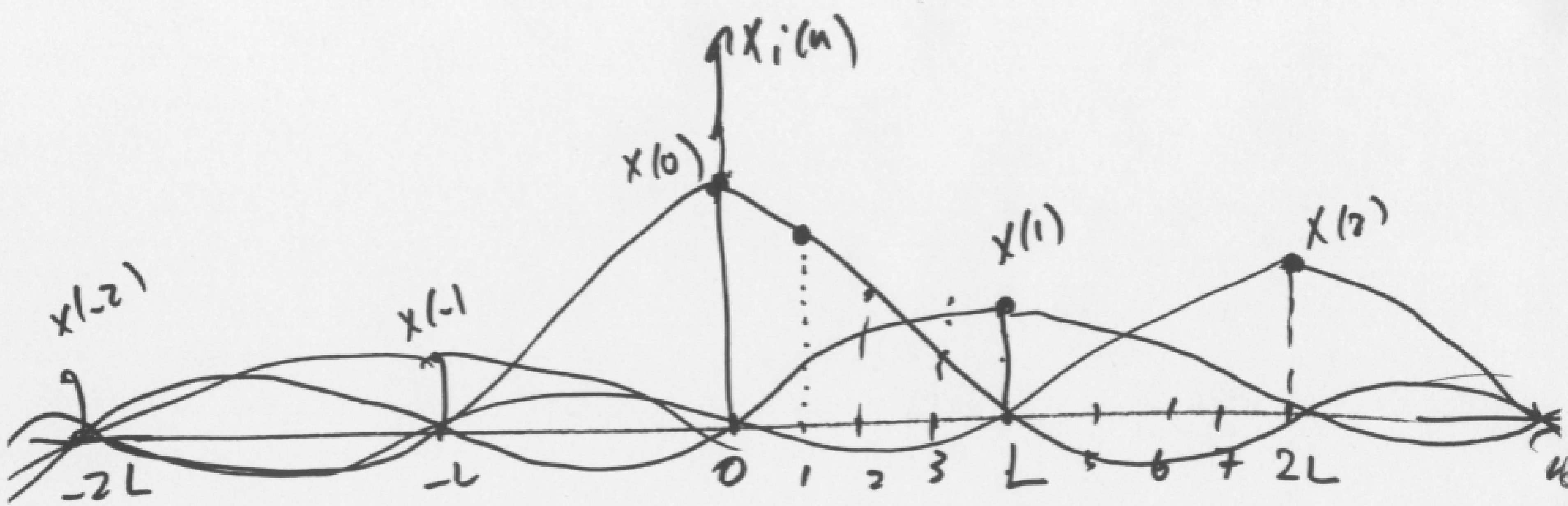
$$h(n) = \frac{\sin \pi n/L}{\pi n/L}$$

plug $x_e(n)$

$$x_i(n) = \sum_k x_e(k)\, h(n-k)$$

$$x_i(n) = \sum_k x(k) \frac{\sin \frac{\pi(n-kL)}{L}}{\pi\left(\frac{n-kL}{L}\right)}$$

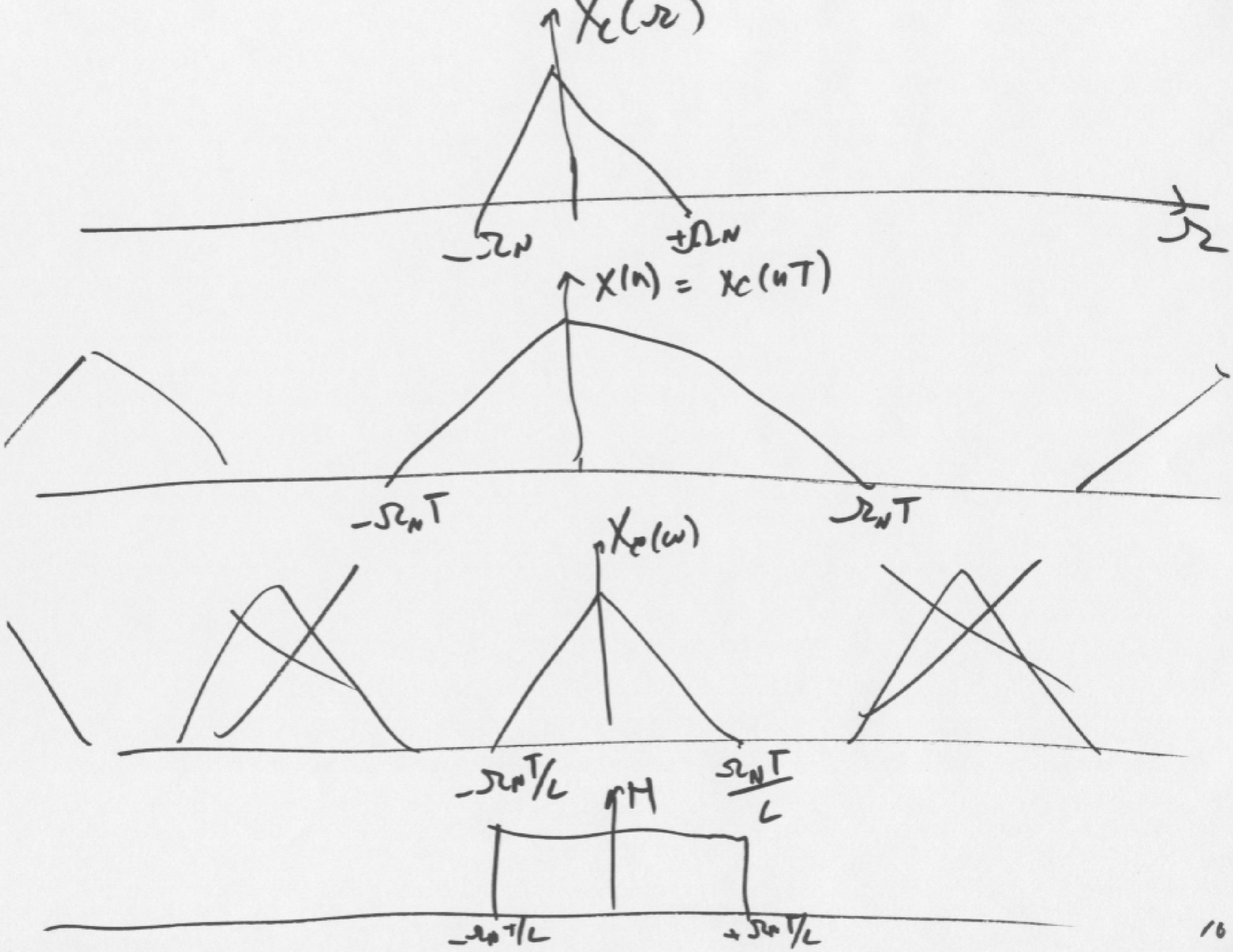
$$x_i(n) = \cdots + x_e(0)\, h(n-0) + 0 + 0 + \ldots \underset{x(1)}{\underbrace{x_e(L)}} h(n-L) + \underset{x(2)}{\underbrace{x_e(2L)}} h(n-2L) + \cdots \cdots$$



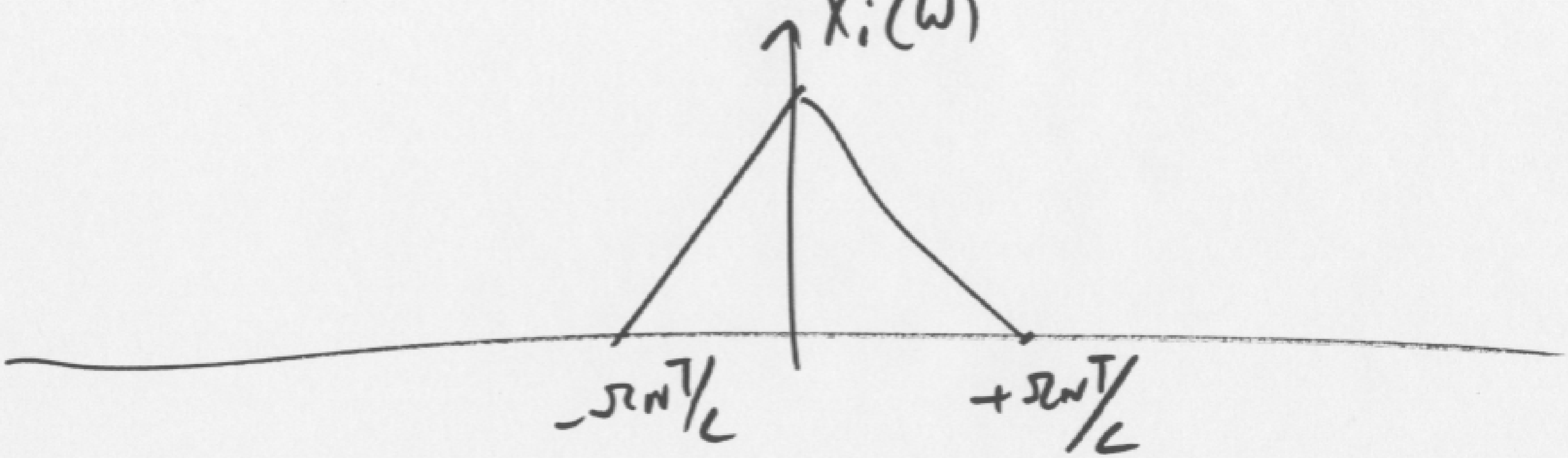
$x_i(n)$
$x(0)$
$x(1)$
$x(2)$
$x(-1)$
$x(-2)$
$-2L$
$-L$
0
1
2
3
L
5
6
7
2L
n

$$x_i(n) = \cdots + x_e(0)\, h(n-0) + x_e(L)\, h(n-L) + x_e(2L)\, h(n-2L) + \cdots$$

$$x_i(n) = \cdots + x(0)\, h(n) + x(1)\, h(n-L) + x(2)\, h(n-2L) + \cdots$$

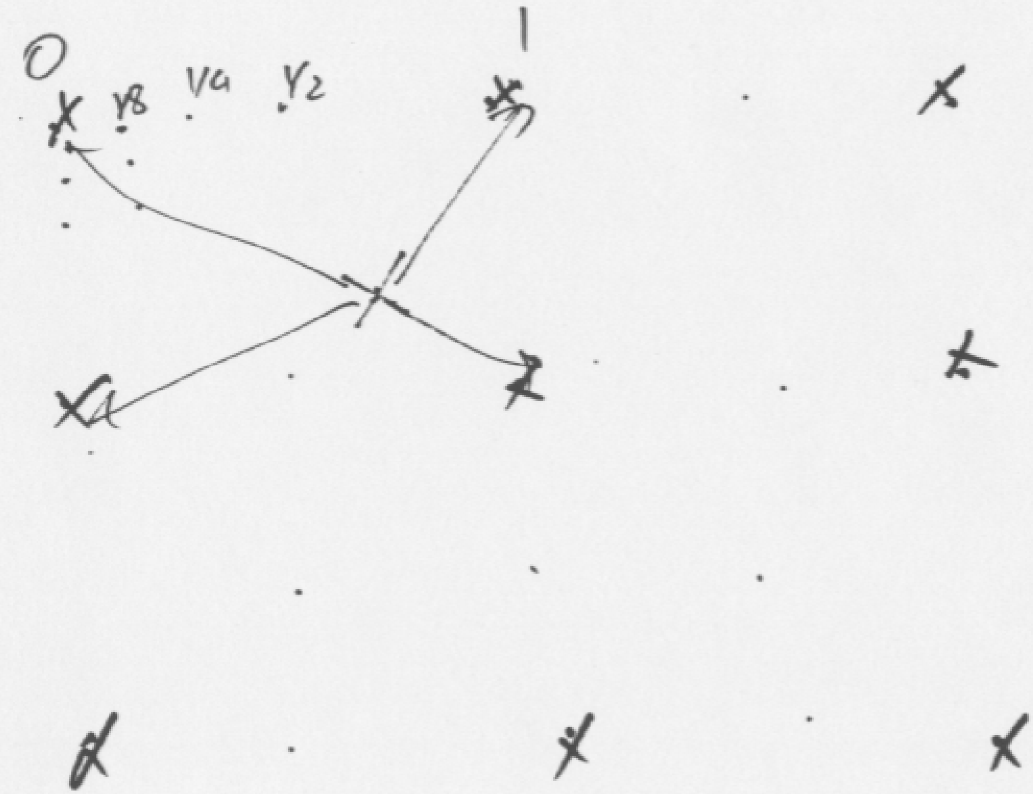


$X_c(\Omega)$
$-\Omega_N$
$+\Omega_N$
Ω
$X(n) = x_c(nT)$
$-\Omega_N T$
$\Omega_N T$
$X_e(\omega)$
$-\Omega_N T/L$
$\frac{\Omega_N T}{L}$
H
$-\Omega_N T/L$
$+\Omega_N T/L$



$X_i(\omega)$
$-\Omega_N T/L$
$+\Omega_N T/L$

Image processing:

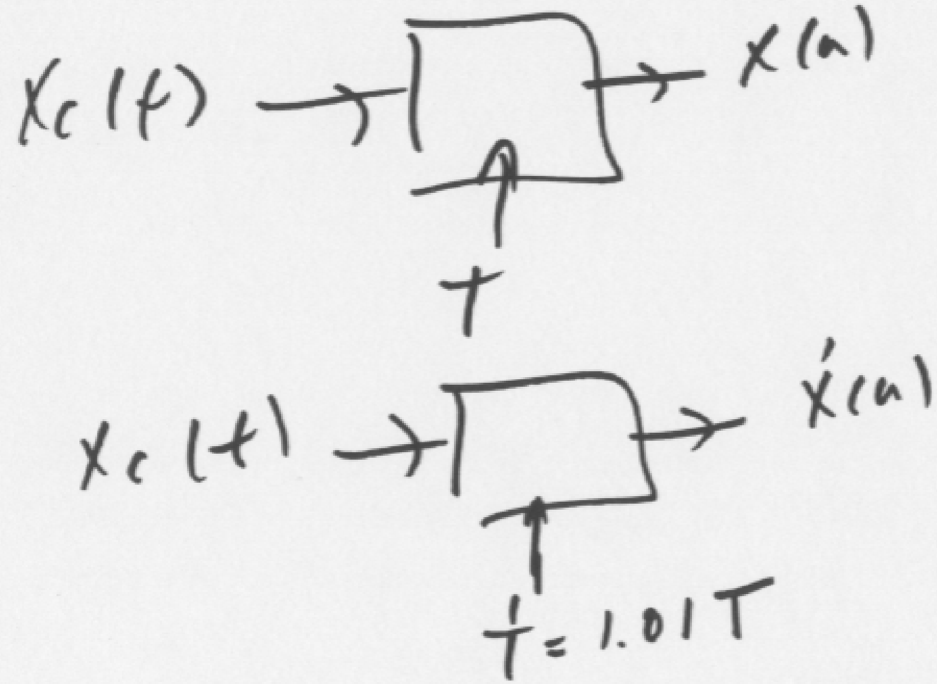


- No Band limited interpolation
- Bilinear
- Bicubic

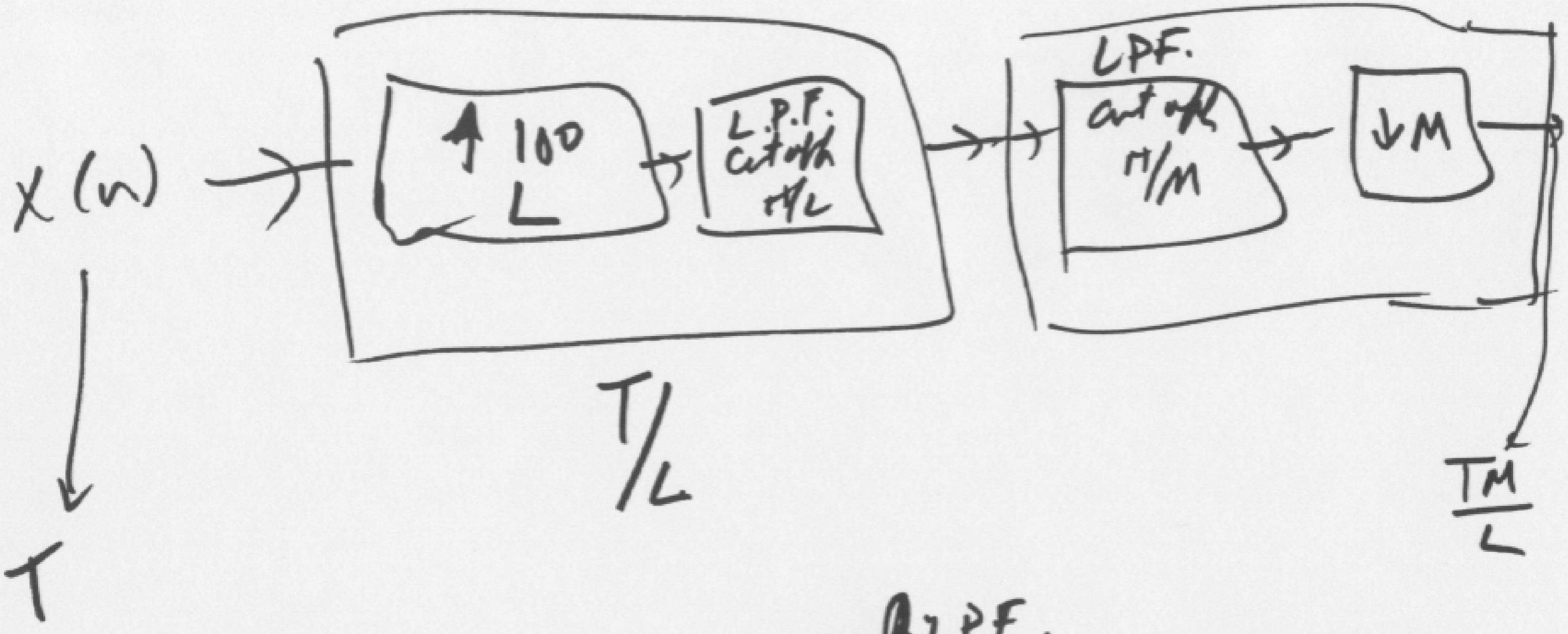
Texture mapping

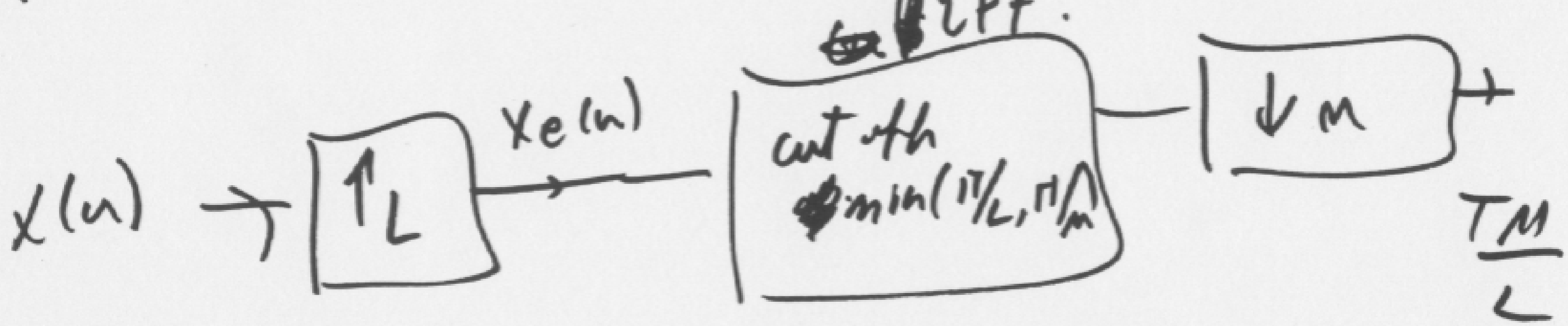
Changing sampling rate by a fraction = non-integer factor.

Change sampling period by fraction.



upsample $L = 100$
downsample $M = 101$


x(n)
↑100
L
L.P.F.
cutoff
π/L
LPF.
cutoff
π/M
↓M
T/L
T
TM/L


x(n)
↑L
xe(n)
LPF.
cutoff
min(π/L, π/M)
↓M
TM/L