Sep 24, 2003 Different realization of LCC DE

Consider a 2nd order D.E.

$$y(n) = a\,y(n-1) + b\,y(n-2) + x(n)$$

(1) $y(n) \leftarrow a\,y(n-1) + b\,y(n-2) + x(n)$

causal.

$H(z)$ ROC outside of some.

(2) $y(n-2) \leftarrow \frac{1}{b}\,y(n) - \frac{a}{b}\,y(n-1) - \frac{1}{b}\,x(n)$

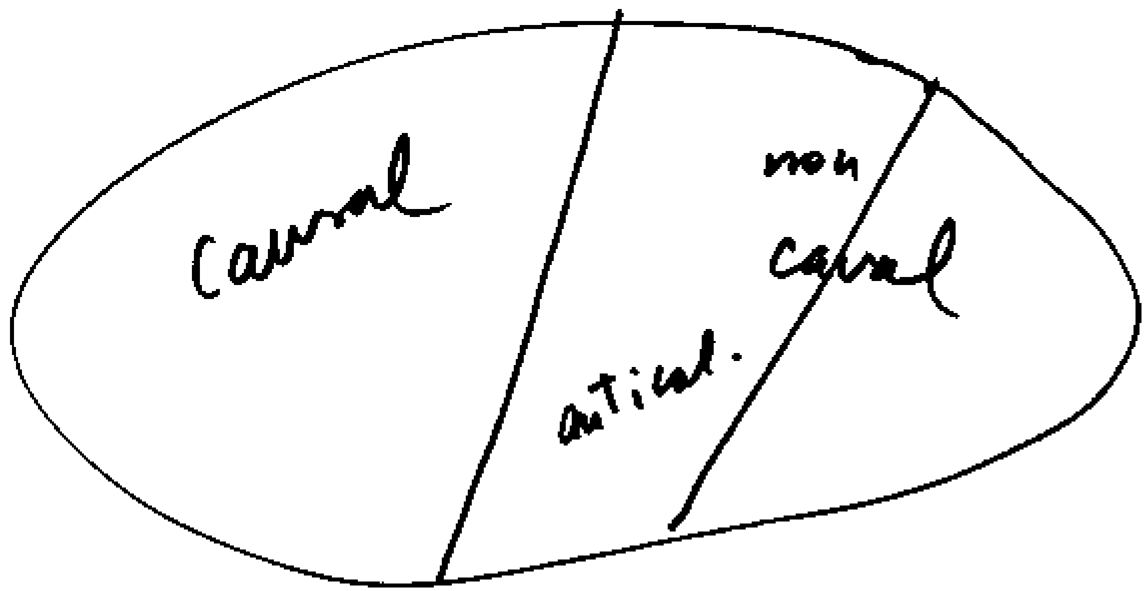
$y(n) \leftarrow \frac{1}{b}\,y(n+2) - \frac{a}{b}\,y(n+1) - \frac{1}{b}\,x(n+2)$

ROC. inside of circle. $\leftarrow$ anti causal implementation.

(3) $y(n-1) \leftarrow \frac{1}{a}\,y(n) - \frac{b}{a}\,y(n-2) - \frac{1}{a}\,x(n)$

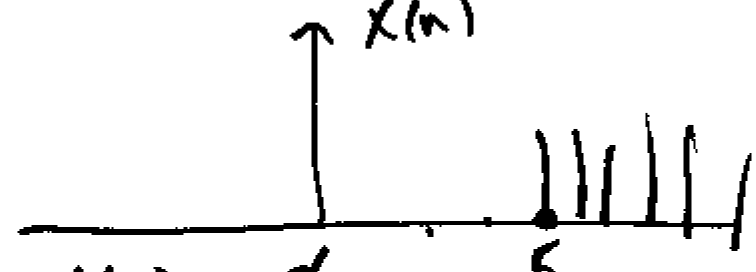
$$y(n) \triangleq \frac{1}{a} y(n+1) - \frac{b}{a} y(n-1) - \frac{1}{a} x(n+1)$$

non causal. ROC: Ring.




IRC. For 2nd order system

(1)

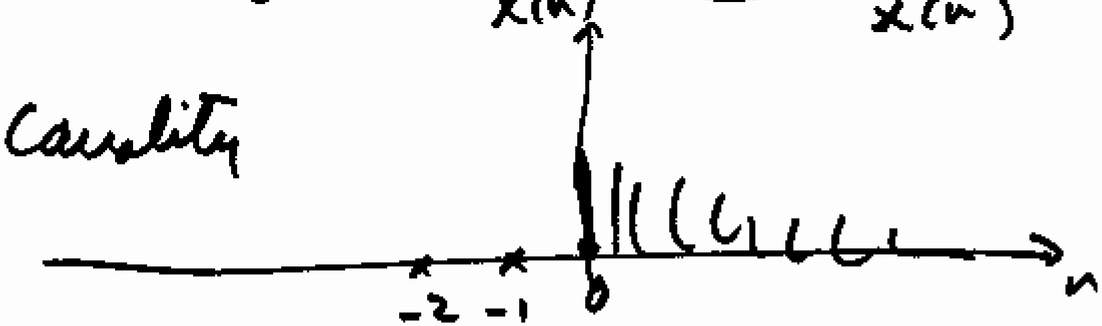



$\Rightarrow$ IRC: $y(3) = y(4) = \phi$

## How To solve L.C.C.D.E w.r.t. IRC or FRC. using Z.T.

Given causal.

**Ex** $y(n) - \frac{5}{6} y(n-1) + \frac{1}{6} y(n-2) = \underbrace{\left(\frac{1}{4}\right)^n u(n)}_{x(n)}$




select I.C. so that LTI & Causality

I.R.C. $\quad y(-1) = y(-2) = \phi$

$$Y(z) - \frac{5}{6} z^{-1} Y(z) + \frac{1}{6} z^{-2} Y(z) = \frac{1}{1 - \frac{1}{4} z^{-1}}$$

$$Y(z) = \frac{6}{1 - \frac{1}{2} z^{-1}} + \frac{-8}{1 - \frac{1}{3} z^{-1}} + \frac{3}{1 - \frac{1}{4} z^{-1}}$$

$\Rightarrow$ ROC chosen to compute I.Z.T outside of "circles"

$$y(n) = 6\left(\frac{1}{2}\right)^n u(n) - 8\,u(n)\left(\frac{1}{3}\right)^n + 3\,u(n)\left(\frac{1}{4}\right)^n$$

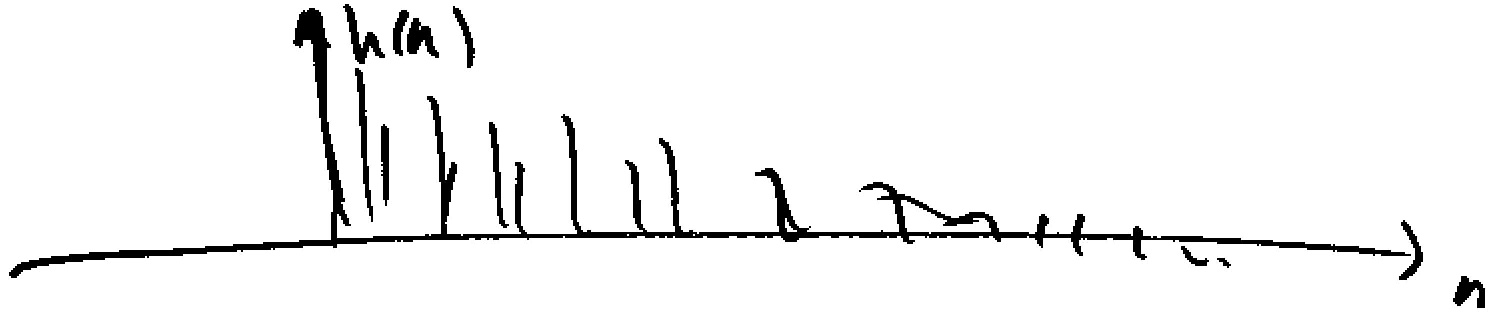
How about Transfer fn?

$$y(n) - \frac{5}{6}\,y(n-1) + \frac{1}{6}\,y(n-2) = \delta(n) = x(n)$$

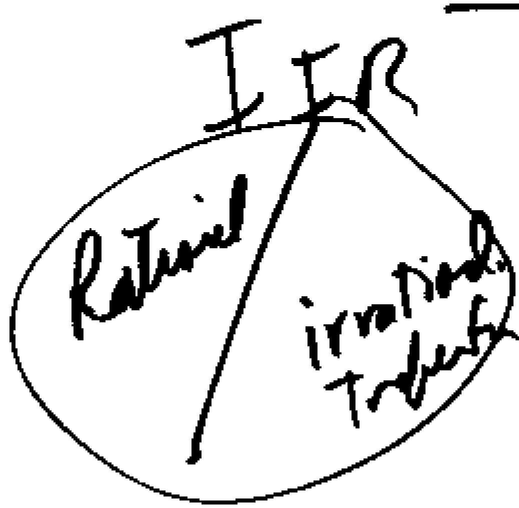
$$\frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}} = H(z)$$

ROC: outside of some circle

$$h(n) = 6\left(\frac{1}{2}\right)^n u(n) - 8\,u(n)\left(\frac{1}{3}\right)^n.$$




## Realization of IIR filters with Rational Transfer fns




Rational T.F $\longrightarrow$ LCCDE

How to realize or implement LCCDE?

$$H(z) = \frac{\sum_{k=0}^{q} b_k z^{-k}}{1 - \sum_{k=1}^{p} a_k z^{-k}} = \frac{Y(z)}{X(z)} = \text{Rational}$$

assume causality

$$y(n) = \sum_{k=1}^{p} a_k \, y(n-k) + \sum_{k=0}^{q} b_k \, x(n-k)$$

Assume $h(n)$ is real. $\rightarrow$ $a_k$, $b_k$ real.

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Notations for flow graph.

(1) $x(n) \circ\!\xrightarrow{\;c\;}\!\circ \; c\,x(n)$

multiplication by a constant $c$

(2) add 2 signals.

$$\begin{array}{l} x(n) \circ\!\searrow \\ \qquad\qquad \circ \; x(n) + y(n) \\ y(n) \circ\!\nearrow \end{array}$$

(3) Delay $\quad x(n) \circ\!\xrightarrow{\;z^{-1}\;}\!\circ \; x(n-1)$

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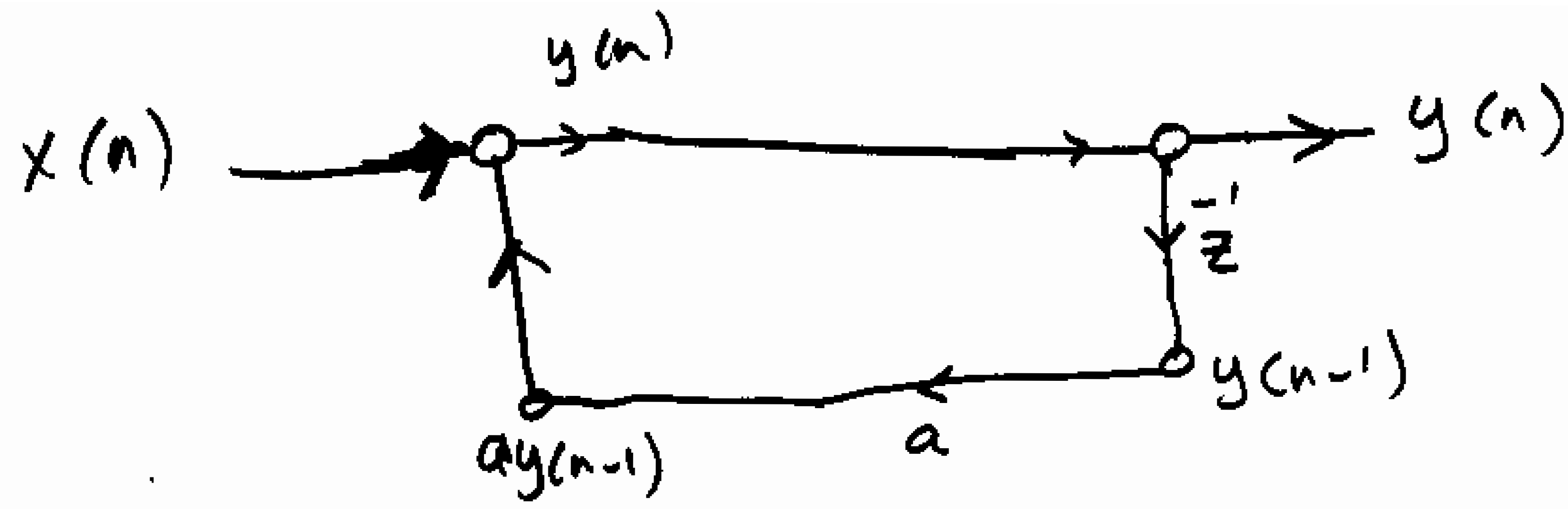
(1) What is flowgraph for this <u>causal</u> system:

$$y(n) = a\,y(n-1) + x(n)$$

$$y(n) \leftarrow a\,y(n-1) + x(n)$$




## Methodolgies for Realizing.

LC.C.D.E.

1. Direct form 1
2. " " 2
3. Cascade
4. Parallel.

Design IIR filter.

1. Specs.
2. Design h(n)
   Rational T.F.
   $\Rightarrow a_k, b_k$ in $H(z)$
3. Realization
4. Implementation
   - C code
   - FPGA
   - ASIC
   - DSP

## Direct Form 1

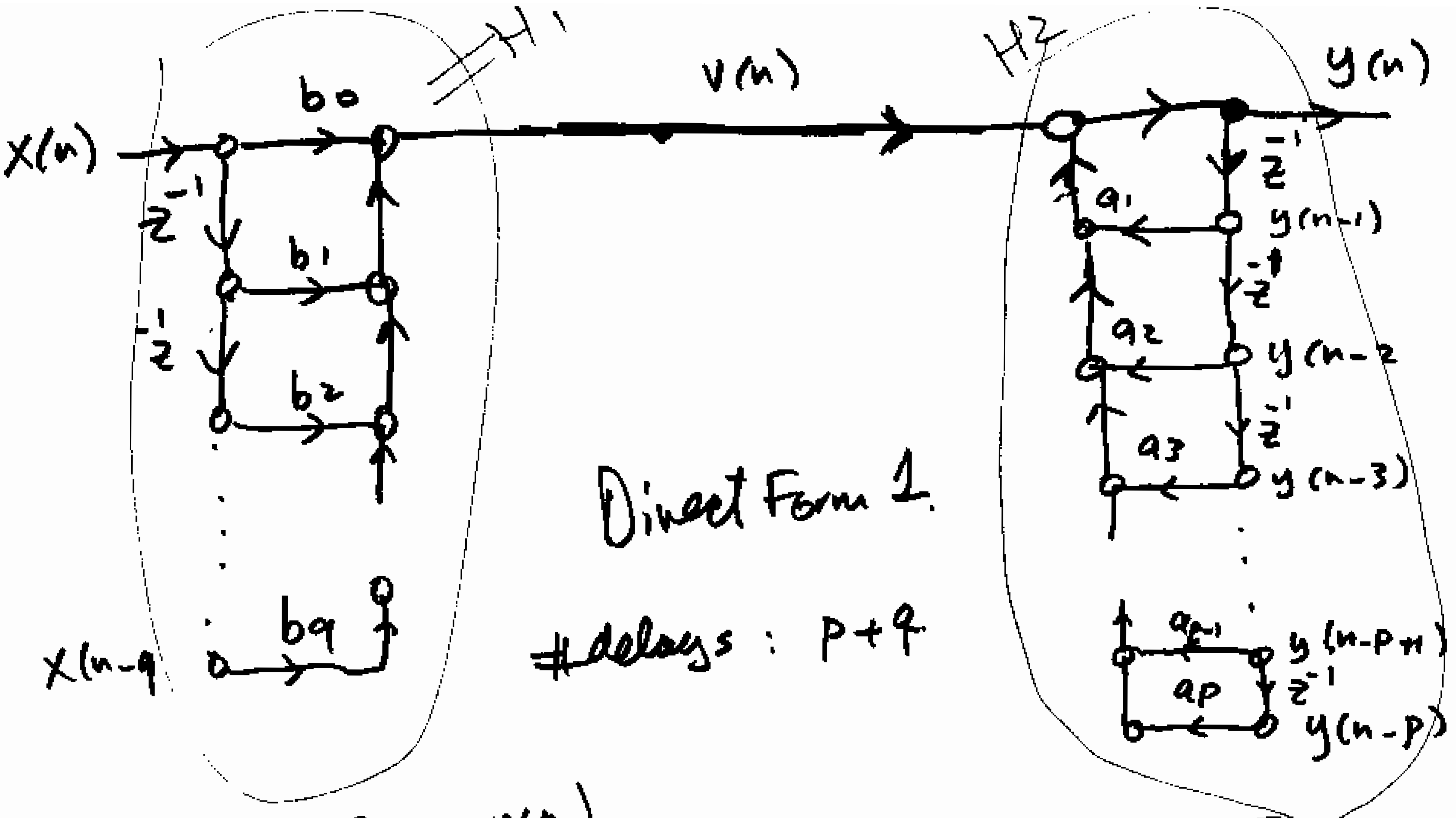
$$y(n) = \sum_{k=1}^{p} a_k\, y(n-k) + \underbrace{\sum_{k=0}^{q} b_k\, x(n-k)}_{v(n)}$$

$b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \ldots\ldots b_q x(n-q)$

$$y(n) = \sum_{k=1}^{p} a_k\, y(n-k) + v(n)$$

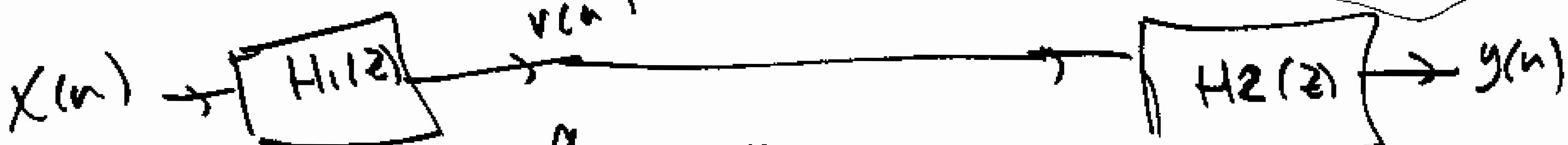
$$y(n) = a_1 y(n-1) + a_2 y(n-2) + \ldots + a_p y(n-p) + v(n)$$

$$Y(z) = \sum_{k=1}^{p} a_k z^{-k} Y(z) + V(z) \Rightarrow \ldots\ldots$$




Direct Form 1.

#delays : p+q




$$H_1(z) = \frac{V(z)}{X(z)} = \sum_{k=0}^{q} b_k z^{-k}$$

$$v(n) = \sum_{k=0}^{q} b_k \, x(n-k)$$

$$\Rightarrow V(z)/X(z) = \sum_{k=0}^{q} b_k z^{-k}$$

$$H_2(z) = \frac{Y(z)}{V(z)} = \frac{1}{1 - \sum_{k=1}^{p} a_k z^{-k}}$$

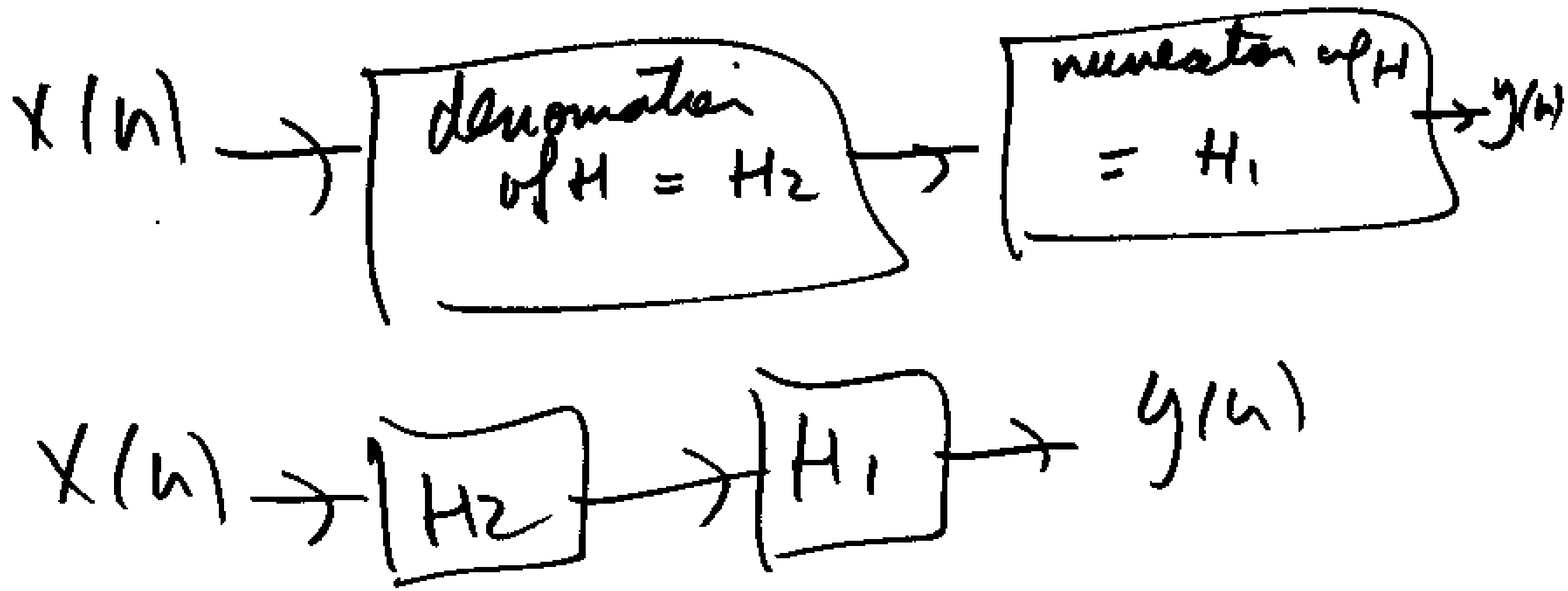
$$X(n) \rightarrow \boxed{H(z)} \rightarrow Y(z)$$

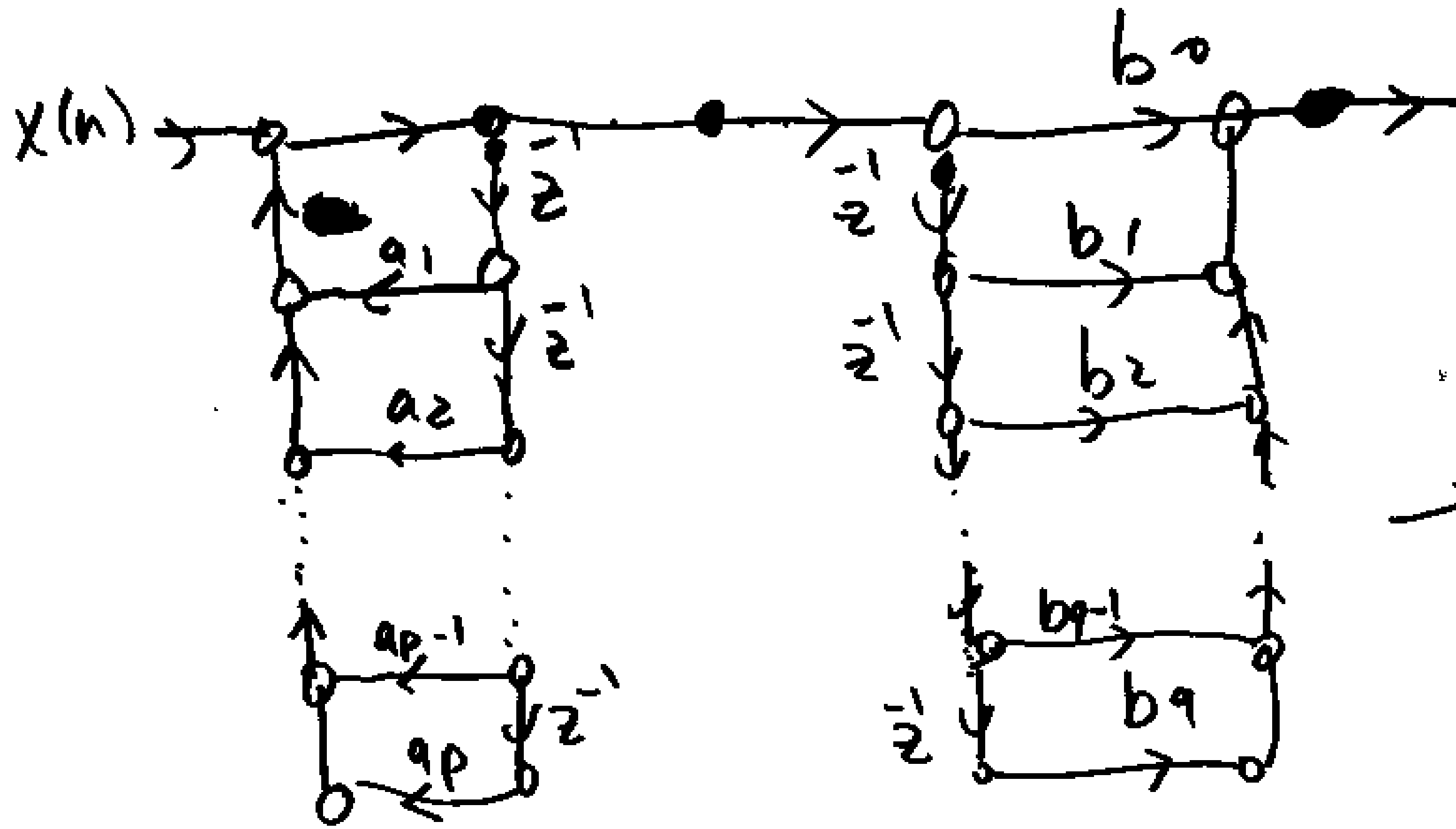
$$H(z) = H_1(z)\, H_2(z) = \frac{\overbrace{\sum_{k=0}^{q} b_k z^{-k}}^{H_1}}{\underbrace{1 - \sum_{k=1}^{p} a_k z^{-k}}_{H_2}}$$

In Direct form 1

$$X(n) \rightarrow \boxed{\text{numerator of } H = H_1} \rightarrow \boxed{\text{denominator of } H = H_2} \rightarrow y(n)$$

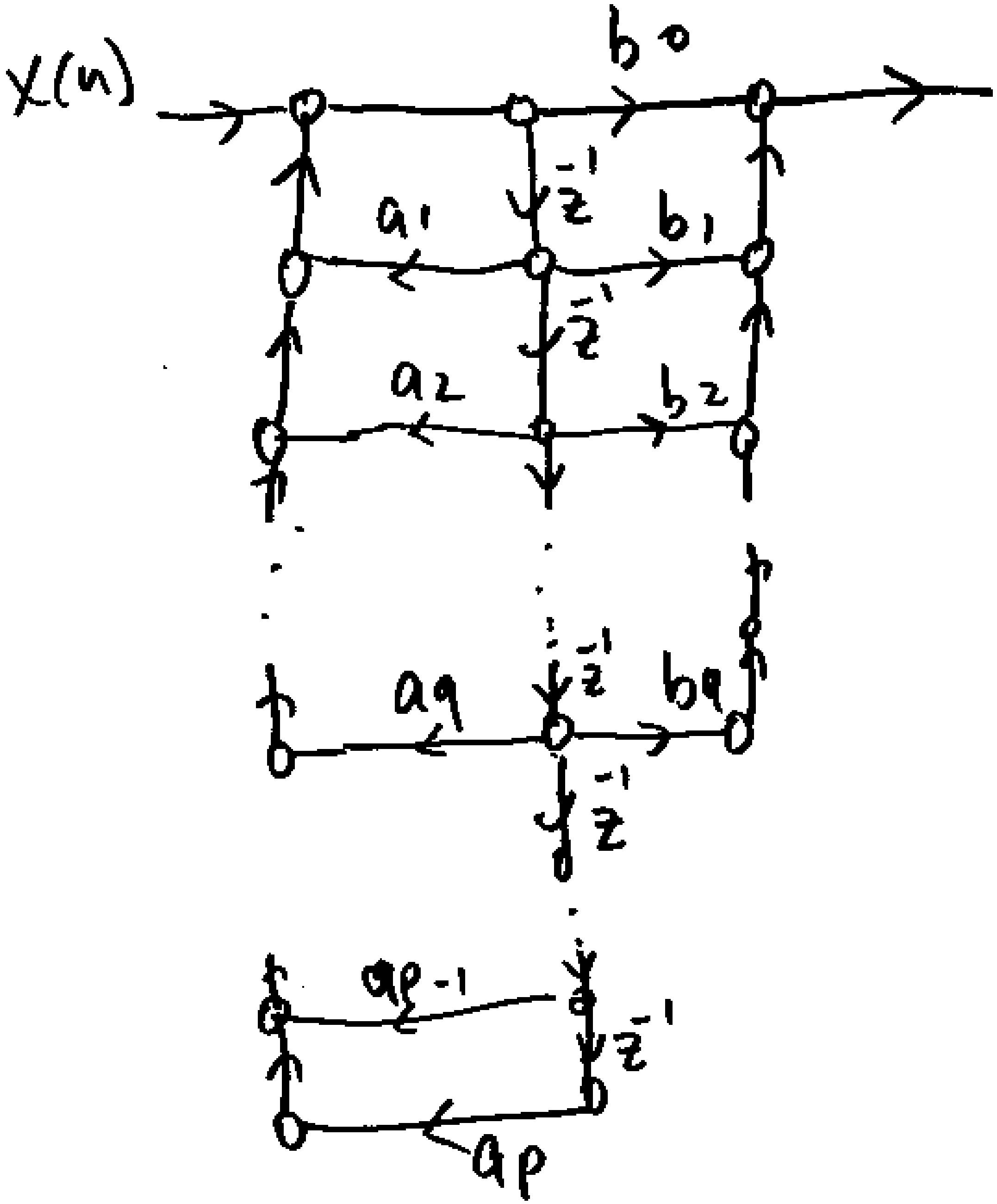
# Direct Form 2






$p > q$

→ collapse $z^{-1}$




Direct form 2.

~~collapsing z~~

# of delays = max (p>q).

lowest # of delays for this system.

Canonical Form