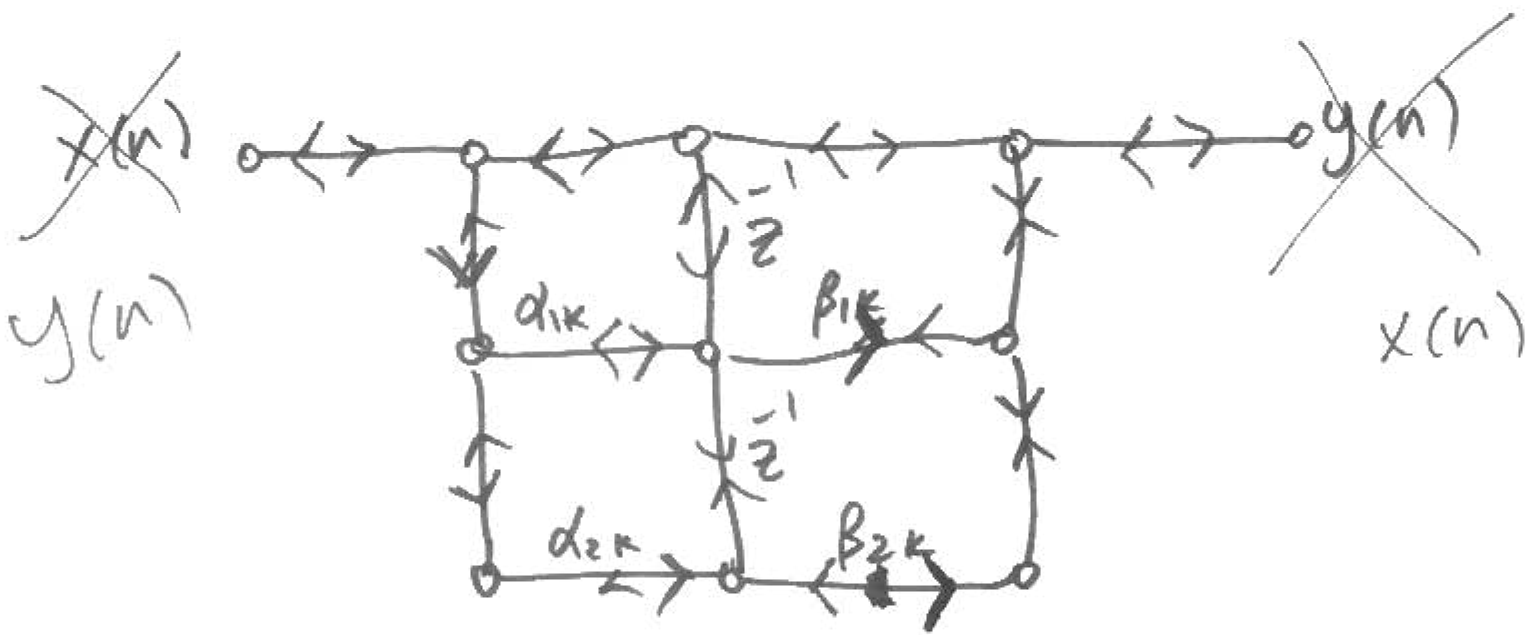
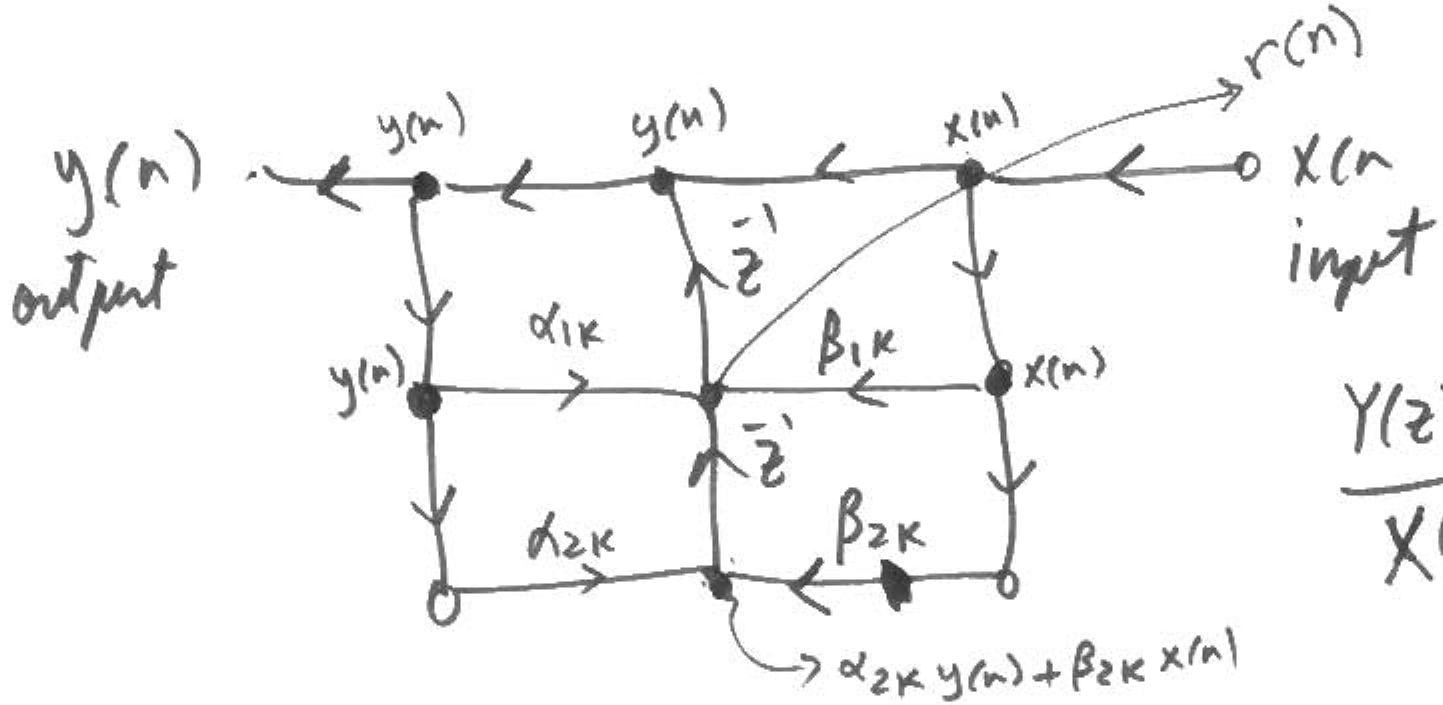
Sep 29, 03

Transposition Thm:

Change order of input/output ; change the direction of flow graph $\Longrightarrow$ get same system: $\Longrightarrow$ save input/output relationship



$$H_k(z) = \frac{1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}}{1 - \alpha_{1k} z^{-1} - \alpha_{2k} z^{-2}}$$



$$\frac{Y(z)}{X(z)}$$

$$\begin{cases} r(n) = \alpha_{1k}\, y(n) + \beta_{1k}\, x(n) + \\ \qquad\qquad \alpha_{2k}\, y(n-1) + \beta_{2k}\, x(n-1) \\ y(n) = x(n) + r(n-1) \end{cases}$$

$$y(n) = x(n) + \alpha_{1k}\, y(n-1) + \beta_{1k}\, x(n-1) + \alpha_{2k}\, y(n-2) + \beta_{2k}\, x(n-2)$$

$$Y(z) = X(z) + z^{-1}\alpha_{1k} Y(z) + z^{-1}\beta_{1k} X(z) + z^{-2}\alpha_{2k} Y(z) + \beta_{2k} z^{-2} X(z)$$

$$\underbrace{\boxed{\frac{Y(z)}{X(z)} = \frac{1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}}{1 - \alpha_{1k} z^{-1} - \alpha_{2k} z^{-2}}}}_{= H_k(z)}$$

Realization Filter

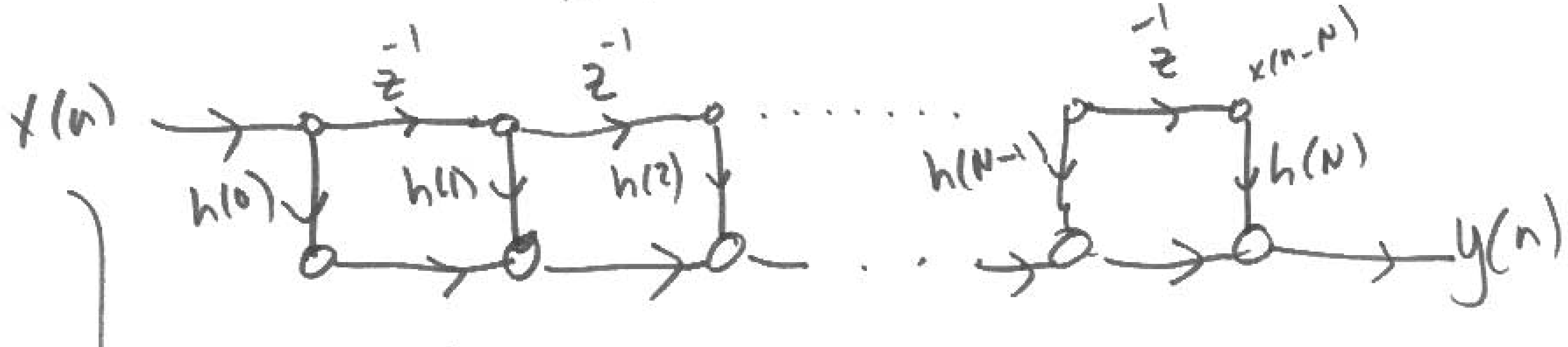
- IIR Rational Transfer fn
  - Direct form 1
  - II
  - Cascade
  - Parallel.
- FIR

$h(n)$ N+1 Taps.

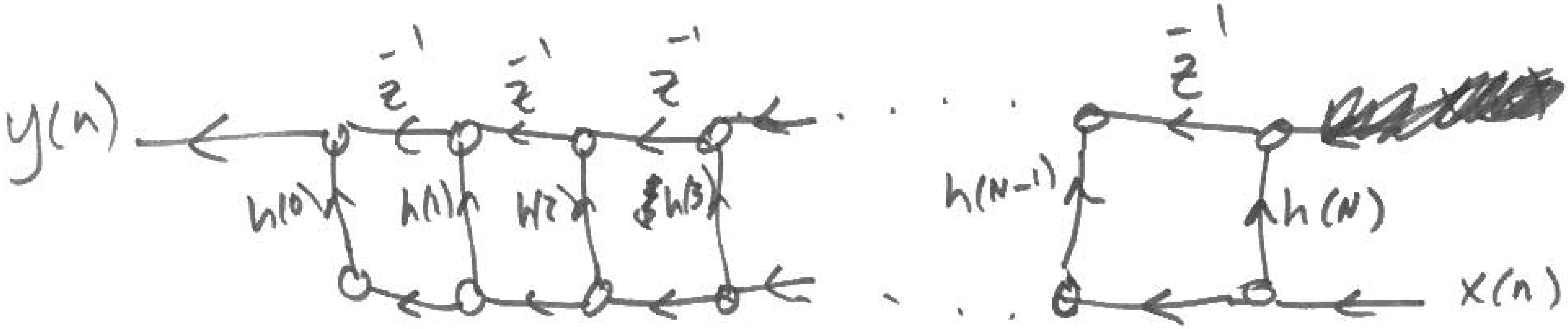
$$\boxed{H(z) = \sum_{k=0}^{N} h(k)\, z^{-k}}$$

$$x \to \boxed{h(n)} \to y$$

$$y(n) = \sum_{k=0}^{N} h(k)\, x(n-k) = h(0)\,x(n) + h(1)\,x(n-1) + h(2)\,x(n-2) + \dots + h(N)\,x(n-N)$$

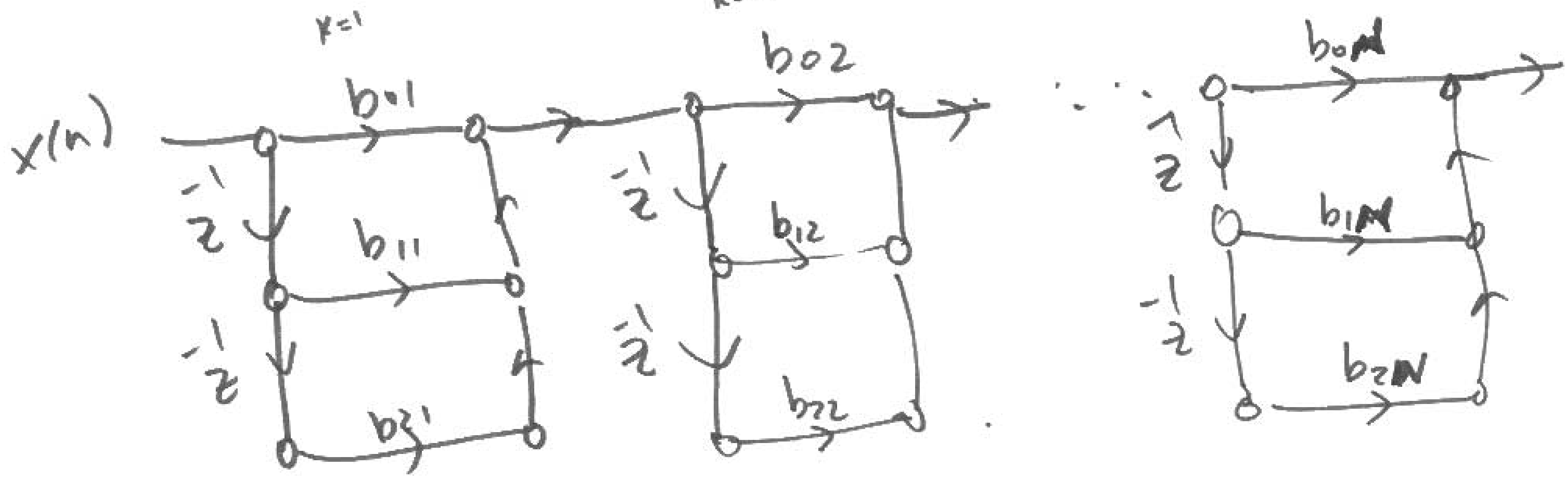


Direct form 1 & II.



Cascade

$$H(z) = \sum_{k=0}^{N} h(k) z^{-k} = \prod_{k=1}^{N} \left(b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2} \right)$$



linear Phase Filtering (LTI)

$$\rightarrow \boxed{\begin{matrix} LTI \\ h(n) \end{matrix}} \rightarrow e^{j\omega n} \quad \underbrace{H(\omega)}$$

$\rightarrow$ DTFT $\{h(n)\}$

= Frequency response filter at ω.

$$\rightarrow \boxed{\begin{matrix} LTI \\ h(n) \end{matrix}} \rightarrow y(n)$$

$$) = X(\omega) H(\omega).$$

$$\omega)| = |H(\omega)| \, |X(\omega)|$$

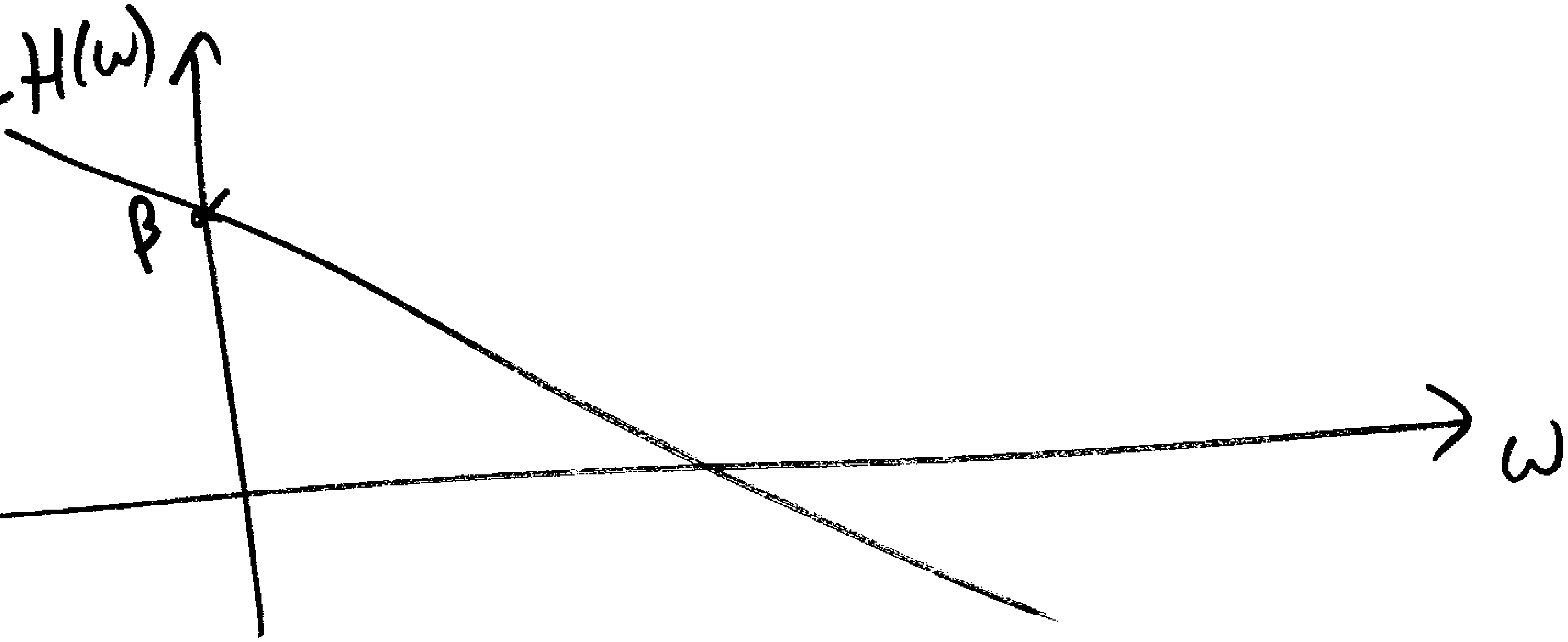
$$Y(\omega) = \measuredangle H(\omega) + \measuredangle X(\omega)$$

$\measuredangle$ = phase.

ear phase $\underbrace{\text{LTI system.}}_{\text{filter.}}$

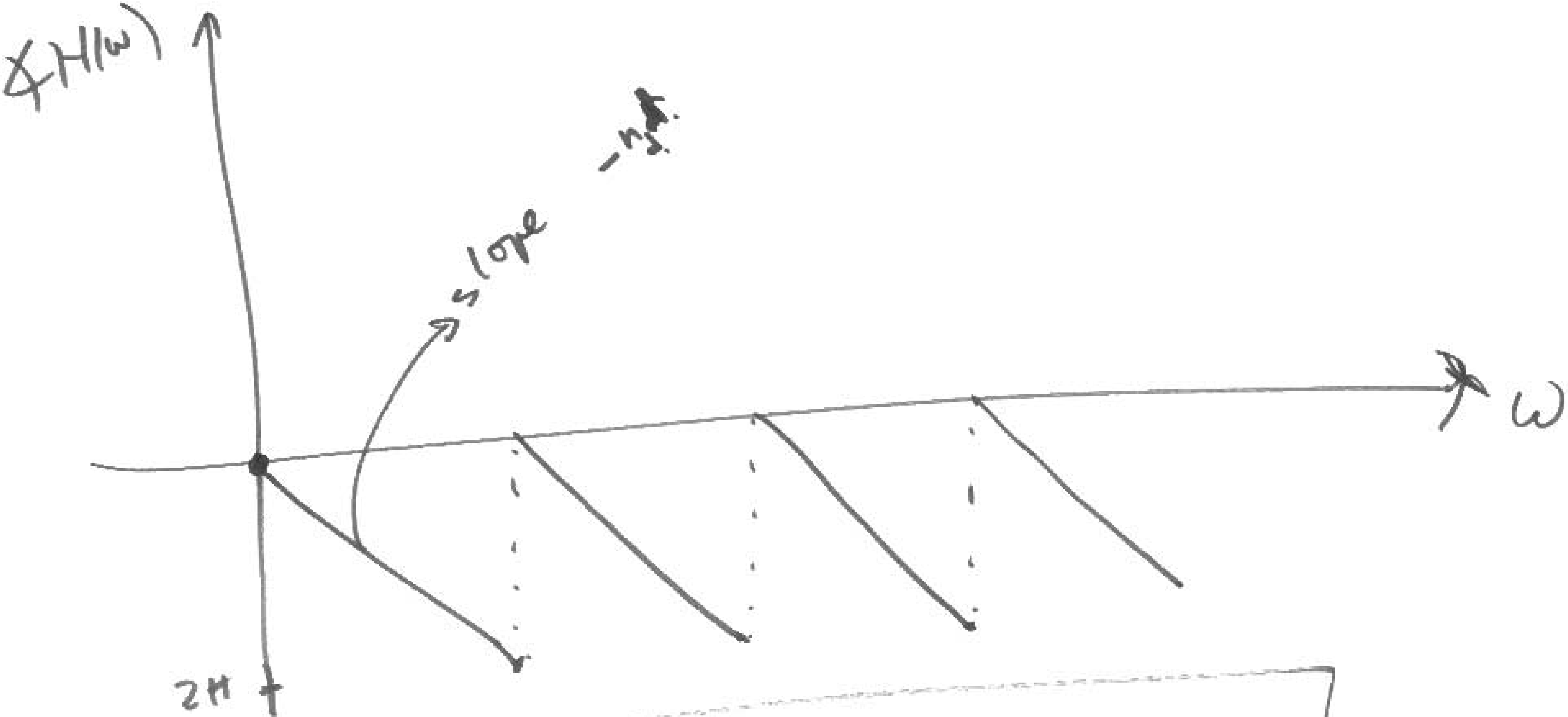
w) is linear in ω

is of the form $\alpha\omega + \beta$



a pure delay: $h(n) = \delta(n - n_d)$

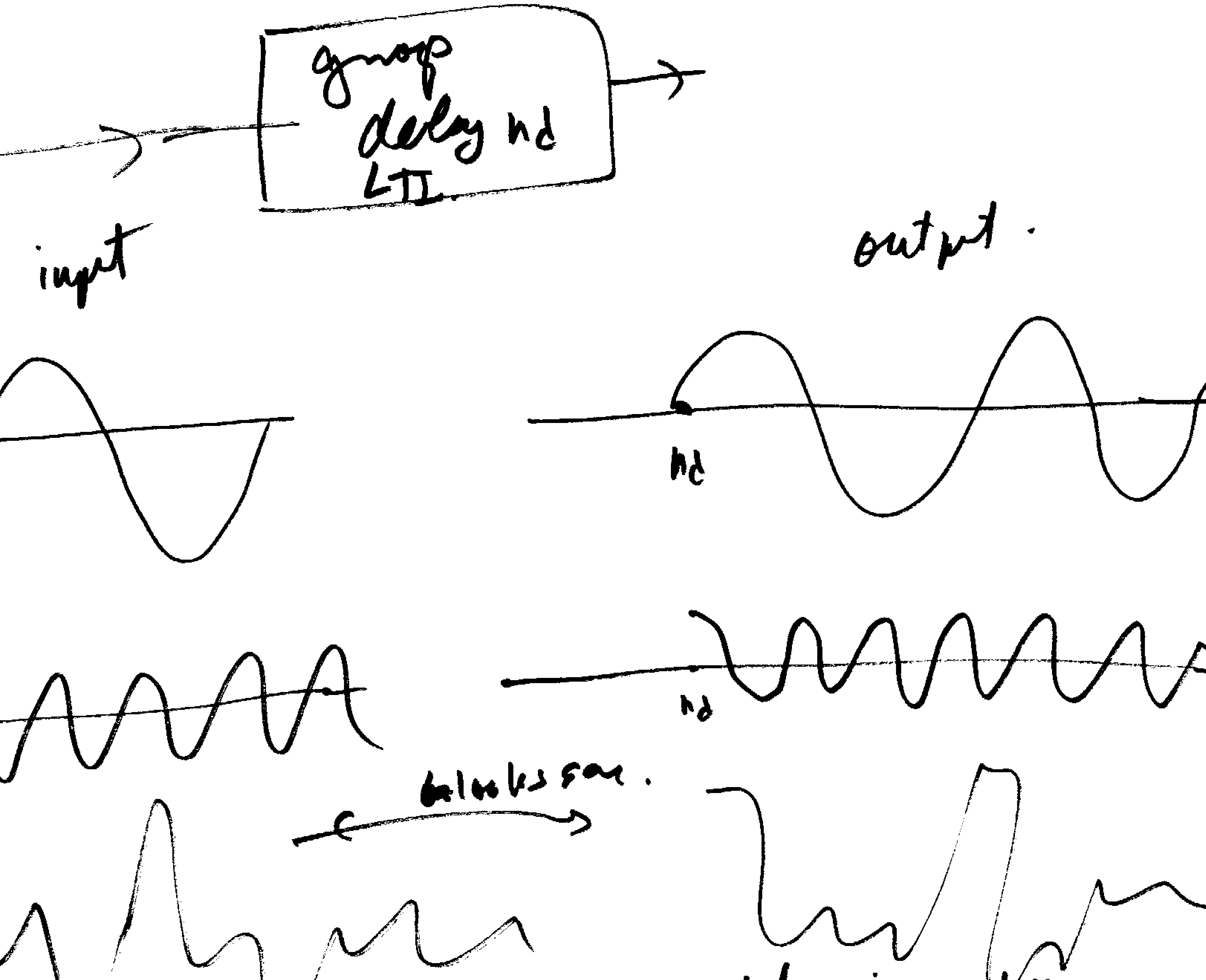
$= 1$ $\quad \measuredangle H(\omega) = -\omega n_d.$



$$\boxed{\text{group delay} \triangleq -\frac{d}{d\omega}\measuredangle H(\omega)}$$

$\measuredangle H(\omega) = -\omega n_d \Rightarrow g.d. = -\frac{d}{d\omega}(-\omega n_d) = n_d$

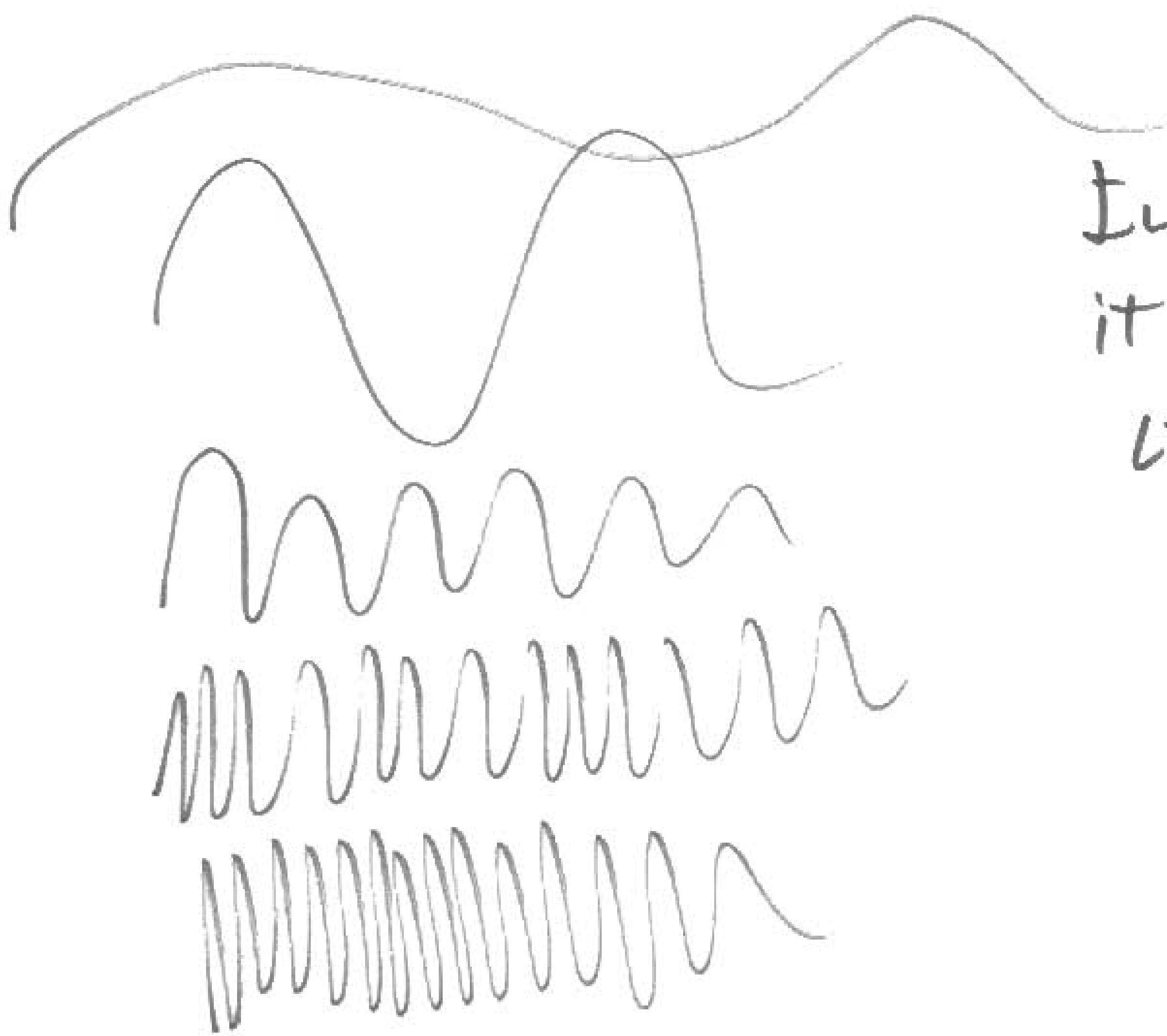
This system delays all sinusoids of different freq by same amount $\leftarrow$ independent of ω.



linear phase $\Longrightarrow$ group delay is constant

$\Longrightarrow$ No distortion is introduced in output

Image



In Image processing application
it helps to have linear phase
LTI system in order to
preserve edge.

Consider a narrowband signal.

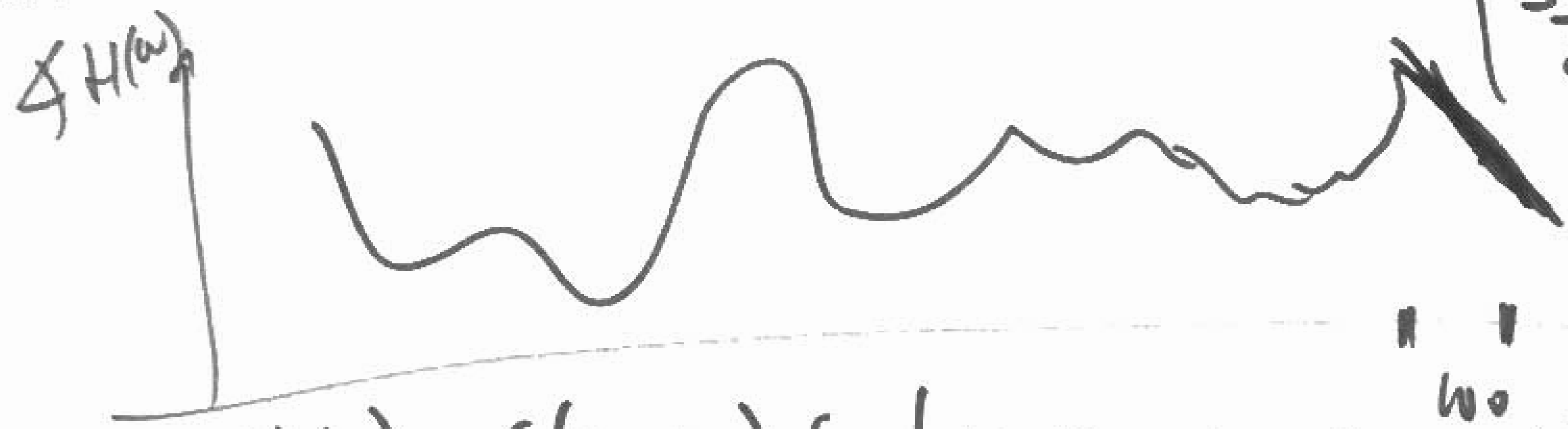
$\Delta\omega \ll \omega_0$

KQED $\omega_0 = 88.5$ MHz.
$\Delta\omega = 0.2$ MHz

$$x(n) = s(n)\cos\omega_0 n$$

- pass $x(n)$ through LTI filter $H(\omega)$ s.Th.
$|H(\omega)| = 1$, can approximate $\measuredangle H(\omega)$ at $\omega = \omega_0$
with a linear Term $\left[\measuredangle H(\omega)\right]_{\omega=\omega_0} = -\phi - \omega n_d$

$$\left[-\frac{d}{d\omega}\measuredangle H(\omega)\right]_{\omega=\omega_0} = n_d$$



Can show: $y(n) = s(n - n_d)\cos(\omega_0 n - \omega_0 n_d - \phi)$

__Conclusion__: For a narrow band signal centered at ω_0

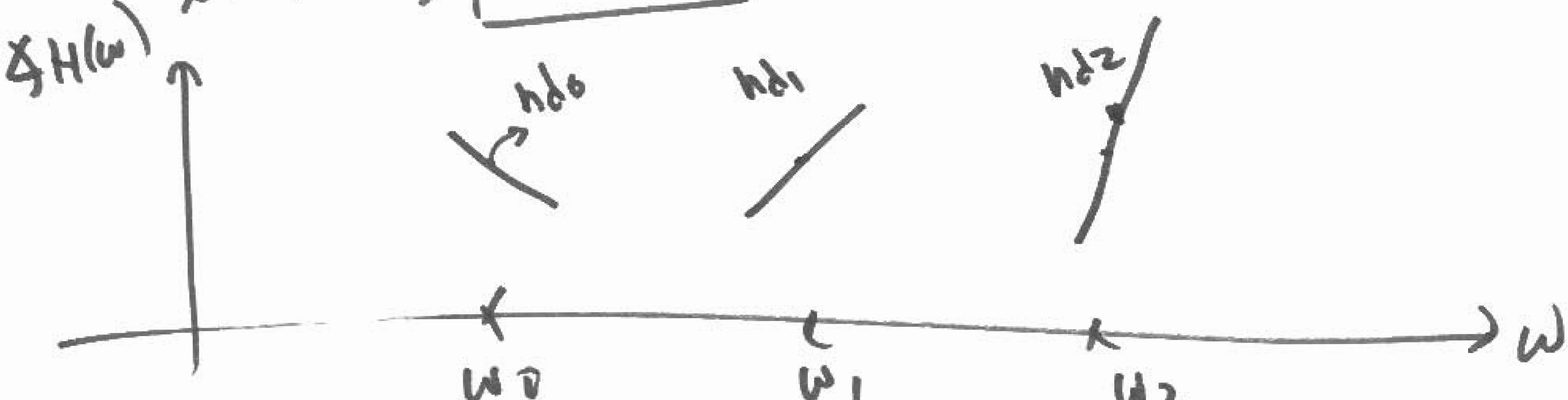
delay is proportional to

$$G_d = \left[-\frac{d}{d\omega} \measuredangle H(\omega) \right]_{\omega = \omega_0}$$

ω_0 = center freq of narrowband signal.

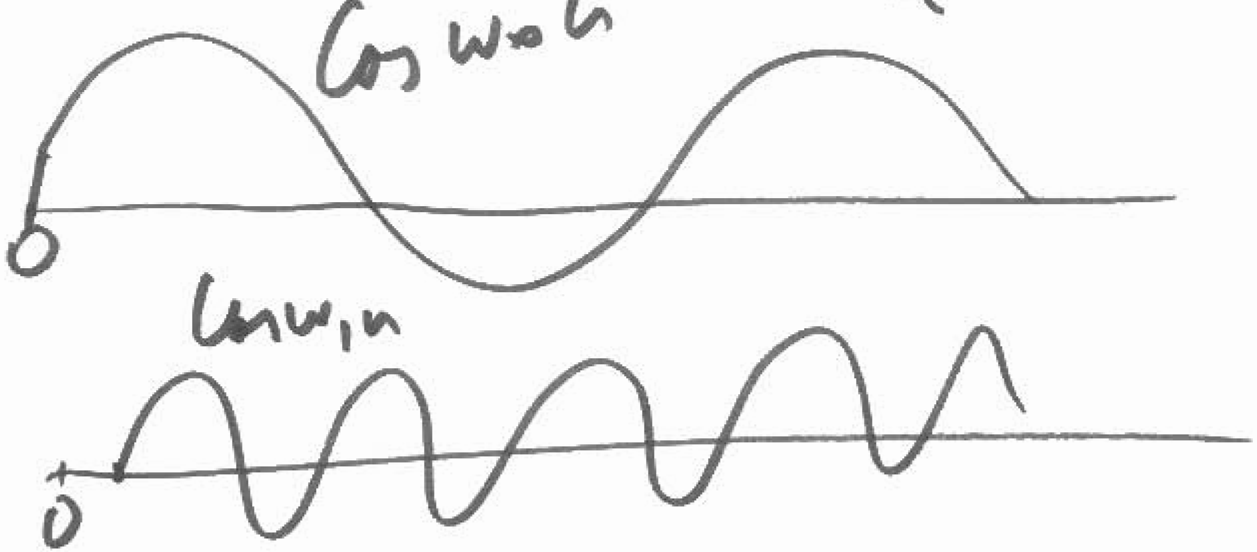
__Consider__: $x(n) = a \cos \omega_0 n + b \cos \omega_1 n + c \cos \omega_2 (n)$

$x(n) \rightarrow \boxed{H(\omega)} \rightarrow y(n)$



$$\begin{cases} a\cos\omega_0 n \\ b\cos\omega_1 n \\ c\cos\omega_2 n \end{cases} \longrightarrow \begin{cases} a\cos(\omega_0 n - \omega_0 n d_0) \\ b\cos(\omega_1 n - \omega_1 n d_1) \\ c\cos(\omega_2 n - \omega_2 n d_2) \end{cases}$$

$$y(n) = a\cos(\omega_0 n - \omega_0 n d_0) + b\cos(\omega_1 n - \omega_1 n d_1) + c\cos(\omega_2 n - \omega_2 n d_2)$$



If $n d_0 = n d_1 = n d_2$
$\Longrightarrow$ no distortion

Otherwise
distorted signal
coming out.