Oct 10, 2013

FIR filter Design Using Windows:

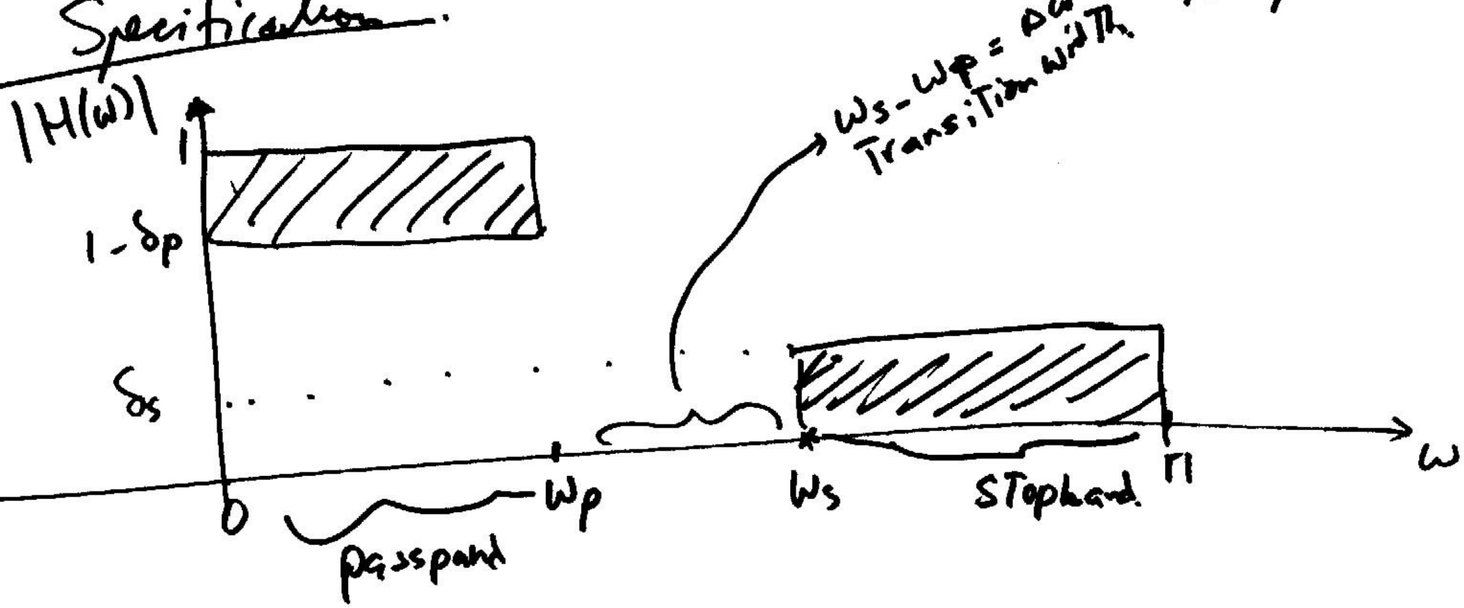
Basic Idea:

1. Ideal filter $H_d(\omega)$
2. Take IDTFT to get $h_d(n)$
3. Multiply Window $h(n) = h_d(n)\, w(n)$ → $H(\omega) = H_d(\omega) * W(\omega)$

$DTFT\{w(n)\} = W(\omega)$

Filter Specification:

low pass.



2,

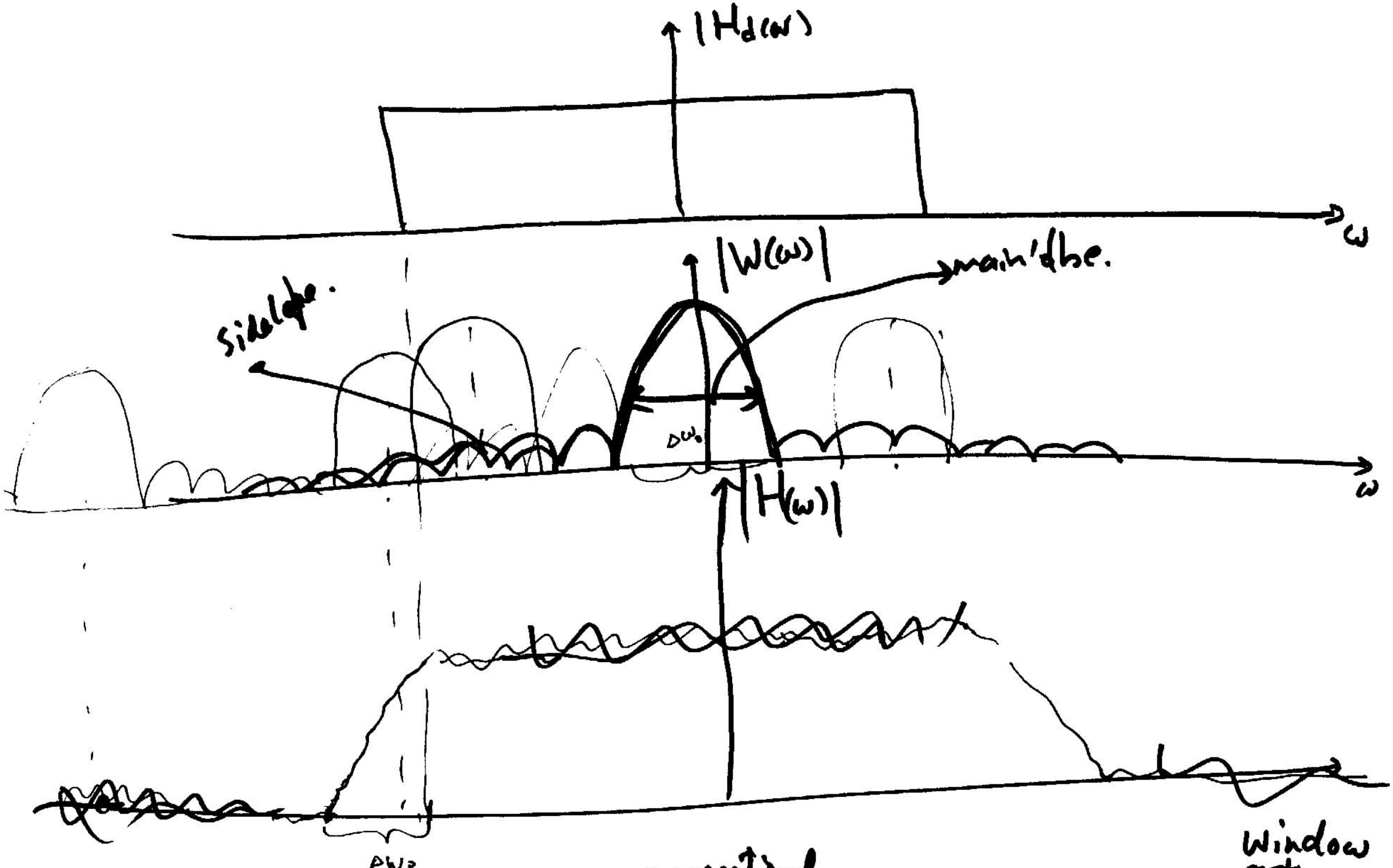
$0 \le \omega \le \omega_p$ Pass band.

$\omega_s \le \omega \le \pi$ stop band

δ_p = passband ripple.

δ_s = stop band ripple.

$\Delta\omega = \omega_s - \omega_p$ = Transition width.



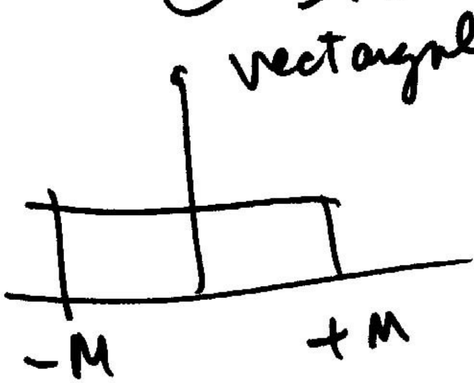
Transition width of $H(\omega)$ ~~depends on~~ proportional. Main lobe width of window
Ripple in passband/stopband proportional To sidelobes of window

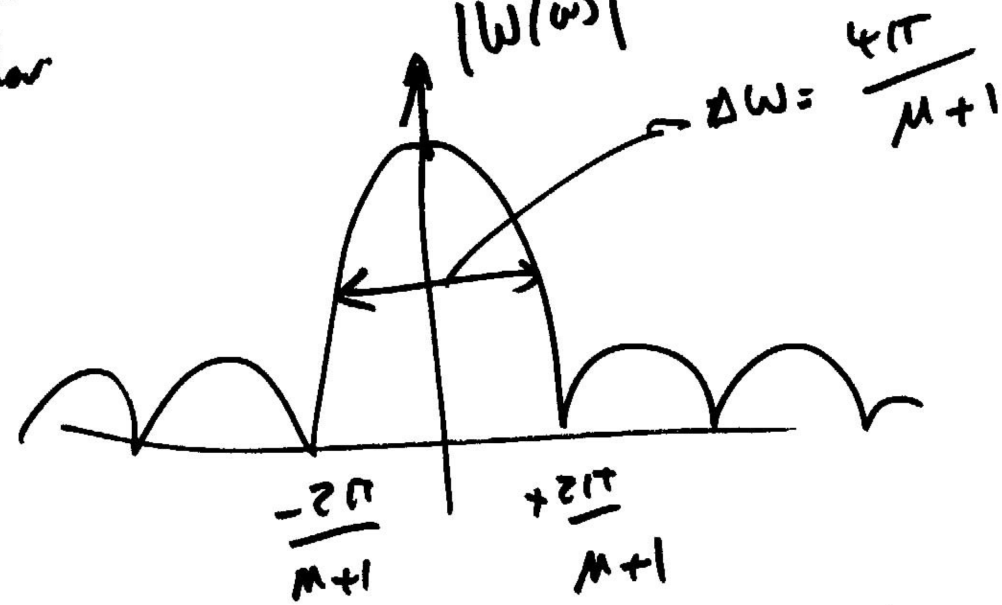
Q: How to design window $w(n)$ to get good mainlobe/sidelobe behavior?

Q1: How to control main lobe width?

ⓐ Size of window. ← Uncertainty principle.

rectangular window





ⓑ Shape of window: Fixed size windows, but different shape have different mainlobe width.

Fig 7.22 O&S.

<u>Q2</u>. How to control sidelobe height?
Shape of window?

Conclusion:

shape → sidelobe
shape → mainlobe

size $\xrightarrow{\text{only}}$ mainlobe.

strategy

① use shape. to control sidelobe.
behavior → controls ripple.

② use size knob to control
choose.
mainlobe behavior → Transition width

Kaiser Window

$$W(n) = \begin{cases} \dfrac{I_0\left[\beta'\left(1-\left(\frac{n-\alpha}{\alpha}\right)^2\right)^{1/2}\right]}{I_0(\beta')} & 0 \le n \le M \\ 0 & \text{otherwise.} \end{cases}$$

I_0 = zeroth order modified Bessel fn.

$$I_0(x) = 1 + \frac{x^2}{2^2(1!)^2} + \frac{x^4}{2^4(2!)^2} + \frac{x^6}{2^6(3!)^2} + \cdots$$

Soln to the following diff equs.

$$x^2\frac{dy^2}{dx^2} + x\frac{dy}{dx} - (x^2+n^2)y = 0$$

$\alpha = 2M$, $N = M+1$ # of Taps

β' effects shape. : Trade off mainlobe width with sidelobe width

Show 7.24 O&S

Design Using Kaiser's Window

① $\Delta\omega = \omega_s - \omega_p$ = Transition width.

② ripple $\delta \longrightarrow A = -20\log_{10}\delta$

Choose $\alpha = M$ and β' as follows.

① $M = 2\alpha = \dfrac{A-8}{2.285\,\Delta\omega}$

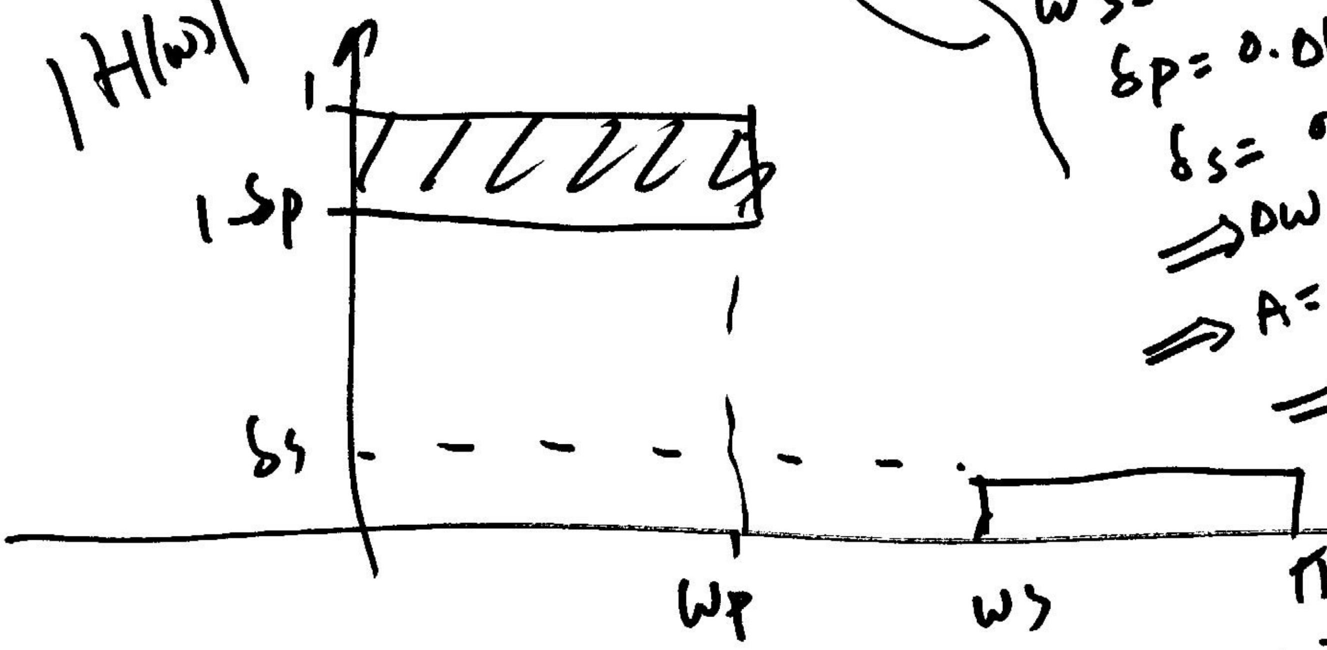
② [illegible]

(2) $\beta' = \begin{cases} 0.1102(A - 8.7) & A > 50 \\ 0.5842(A - 21)^{0.4} & 21 \le A < 50 \\ 0 & A < 21 \end{cases}$

$H_d(\omega) = |H_d(\omega)| e^{j\beta} e^{j\alpha\omega}$

$\beta = 0 \Rightarrow$ Type I, or type II

Ex LPF Specs



$\omega_p = 0.4\pi$

$\omega_s = 0.6\pi$

$\delta p = 0.01$

$\delta s = 0.001$ $\Rightarrow \delta = 0.001$

$\Rightarrow \Delta\omega = 0.2\pi$

$\Rightarrow A = -20\log_{10}\delta = -60$

$\Rightarrow$ plug in $\Rightarrow$

$\Rightarrow M, \beta'$

$\Rightarrow M = 37$

$\Rightarrow$ # Taps $= 38.$

$\Rightarrow$ Type II

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