Oct 15, 03

Optimal FIR filter Design

Type I: generalized linear phase filter.

LPF

$h(n) \leftarrow H(\omega) = H_m(\omega)\, e^{j\beta}\, e^{-j\alpha\omega}$ $\leftarrow$ generalized linear phase.

type I: $\beta = 0$: N # of taps is odd $= 2M+1$

$h(n) = h(N-n-1)$

$$H_m(\omega) = \sum_{n=0}^{M} a(n) \cos(\omega n) \triangleq G(\omega)$$

$a(0) = h(0)$

$a(n) = 2h(M-n)$ all other n.

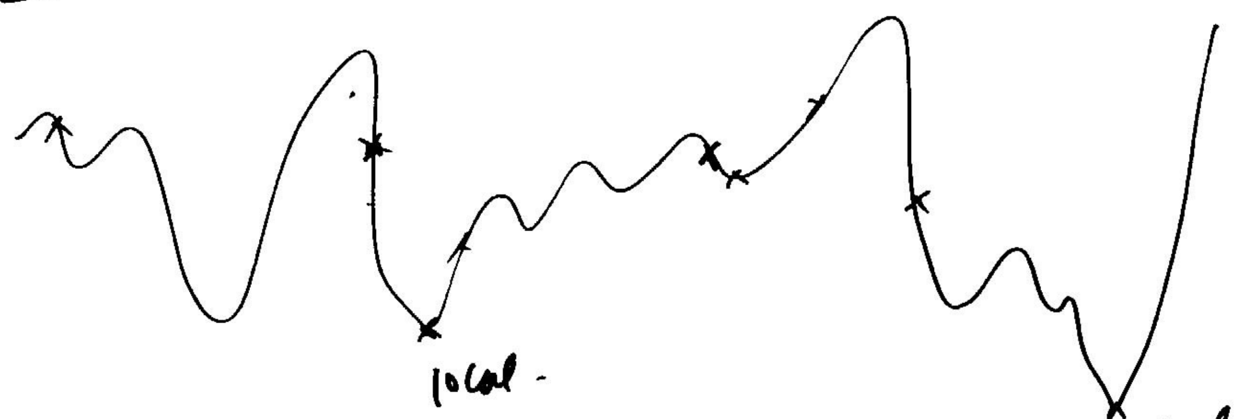
$\triangleq G(\omega)$

Observations on $G(\omega)$:

(1) $G(\omega)$ is a continuous fn of ω, and is as many times differentiable as you want.

(2) **Q**: How many local extrema does $G(\omega)$ have?

extrema → minima
→ maxima.



Express $\cos(\omega n)$ as sum of powers of $\cos\omega$.

$$\cos 2\omega = 2\cos^2\omega - 1$$

$$\cos 3\omega = \cos(2\omega + \omega) = \cos 2\omega \cos\omega - \sin 2\omega \sin\omega$$

$$= \cos\omega\left[2\cos^2\omega - 1\right] - 2\sin^2\omega\cos\omega$$

$$= 2\cos^3 w - \cos w - 2\cos w\left[1 - \cos^2 w\right]$$

$$= 2\cos^3 w - \cos w - 2\cos w + 2\cos^3 w$$

$$\cos 3w = 4\cos^3 w - 3\cos w$$

General : $\cos(wn)$ as sum of powers of $\cos w$.

$$\cos(wn) = \sum_{i=0}^{n} \alpha_i (\cos w)^i$$

α_i → Tchebychev coefficient

$$G(w) = \sum_{n=0}^{M} a(n)\left[\sum_{i=0}^{n} \alpha_i (\cos w)^i\right]$$

$$G(w) = \sum_{n=0}^{M} \gamma(n)\,(\cos w)^n$$

To compute local extrema, Take derivate set equal to zero.

$$\frac{dG(\omega)}{d\omega} = 0 \Rightarrow \sum_{n=0}^{M} \gamma(n)\, n \left(\cos\omega\right)^{n-1} \left(-\sin\omega\right) = 0 \Rightarrow$$

$$\left(-\sin(\omega)\right) \sum_{n=0}^{M} n\gamma(n) \left(\cos\omega\right)^{n-1} = 0$$

$$\Rightarrow \begin{cases} \sin\omega = 0 \Rightarrow \omega = 0, \pi \\ \displaystyle\sum_{n=0}^{M} \gamma(n)\, n \underbrace{\left(\cos\omega\right)}_{\cos\omega = x}{}^{n-1} = 0 \Rightarrow \text{Max of } M-1 \text{ zeros.} \end{cases}$$

$$\Rightarrow \boxed{\text{Total \# of local extrema is } (M-1) + 2 = M+1}$$

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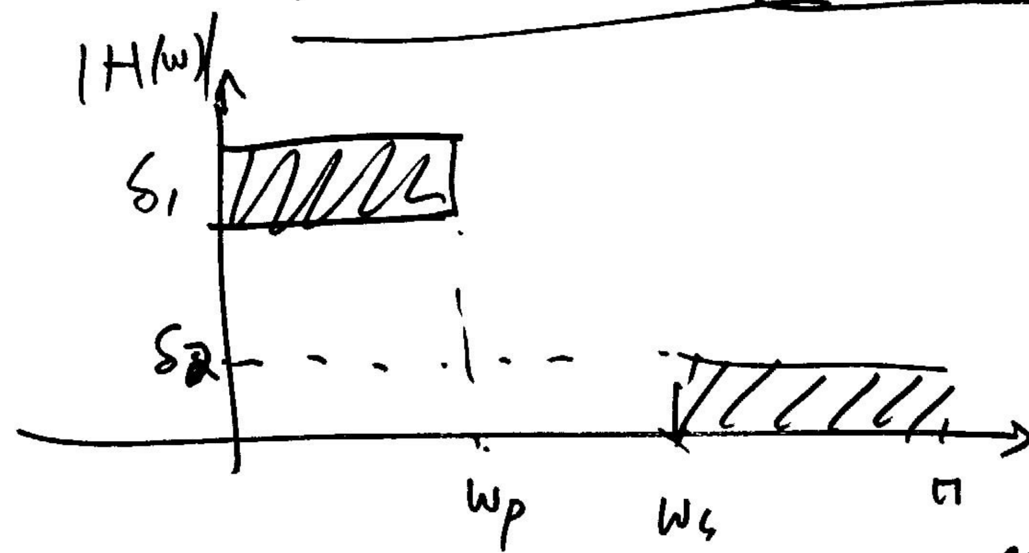
Problem Statement Optimal FIR Filter Design

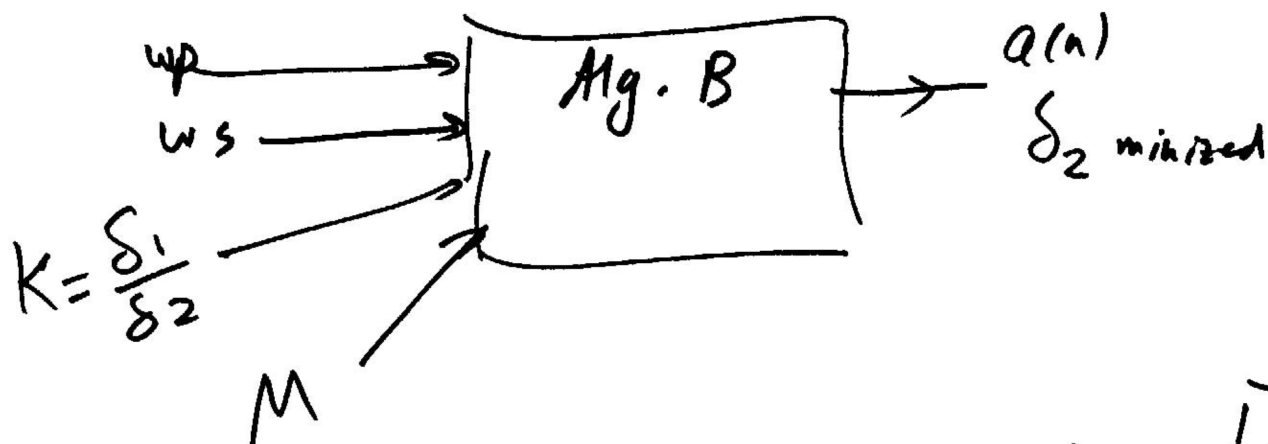
Problem A:



Given ω_s, ω_p, δ_1, δ_2 Determine coeff of $G(\omega)$, i.e $a(n)$ such that M is minimized

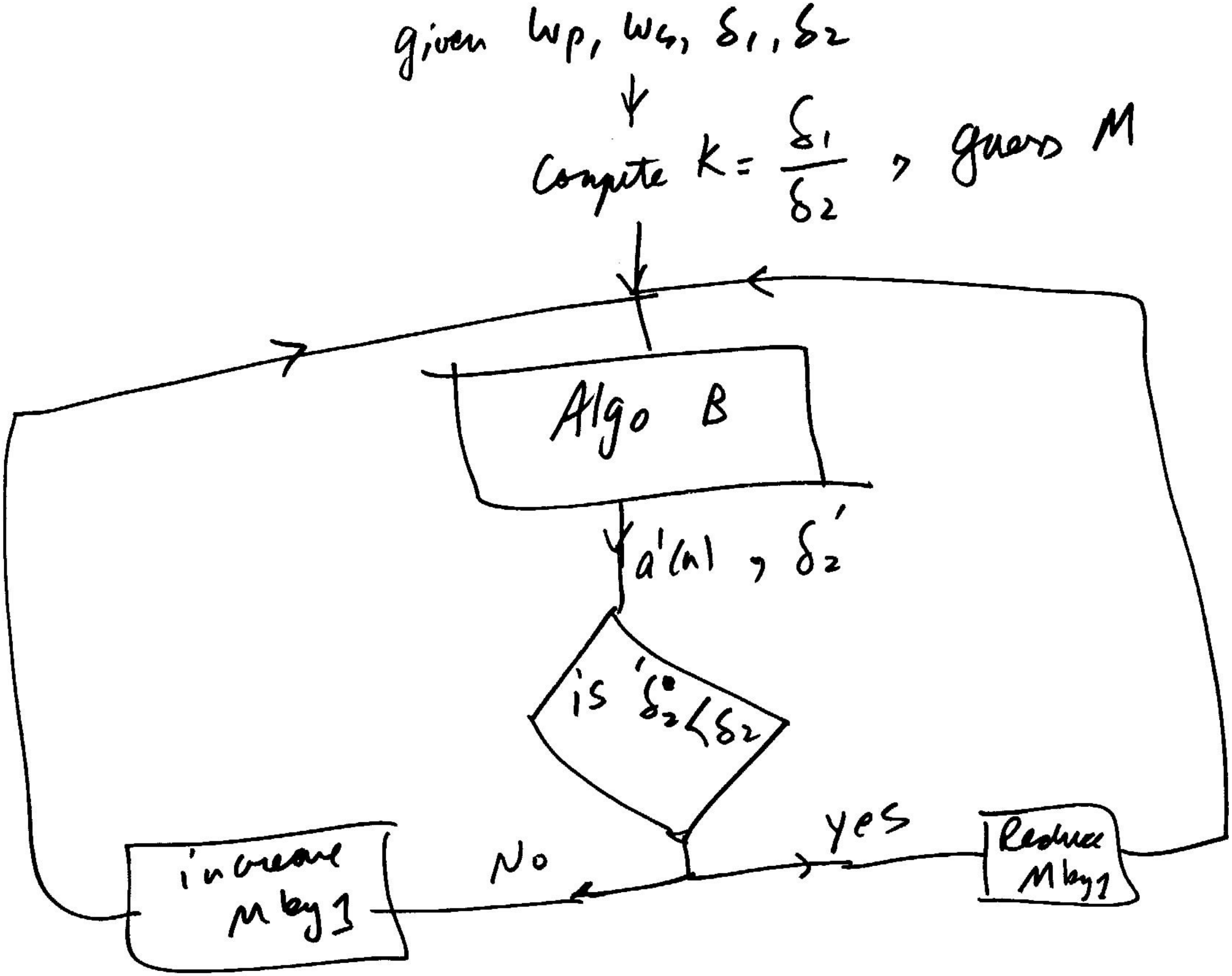
Problem B Given wp, ws, M, $K = \frac{\delta_1}{\delta_2}$

Determine $a(n)$ such that δ_2 is minimized.



Show that if I had a box $\boxed{\text{Algo B}}$ box

Then I can use it, to solve Problem A.


given w_p, w_s, δ_1, δ_2
Compute $K = \frac{\delta_1}{\delta_2}$, guess M
Algo B
$a'(n)$, δ_2'
is $\delta_2' < \delta_2$
No
yes
increase M by 1
Reduce M by 1