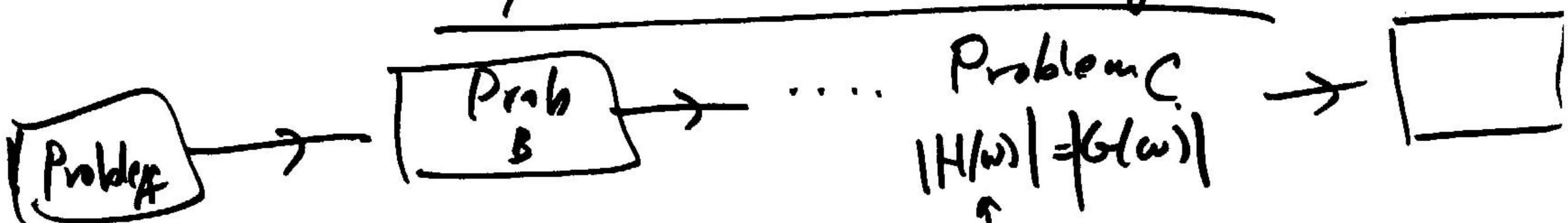
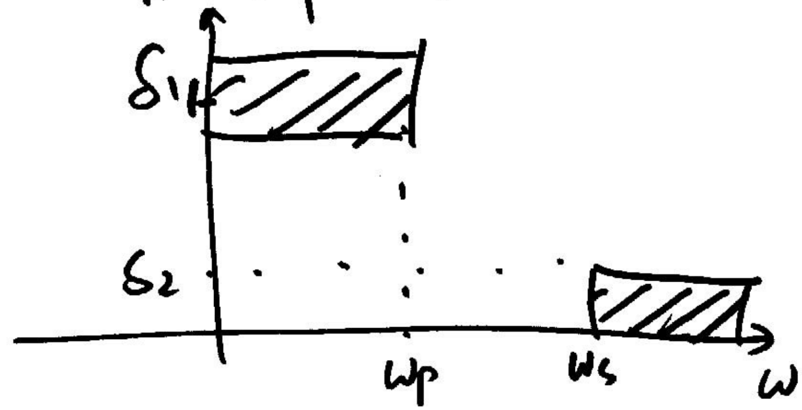
oct 16, 03

Optimum FIR filter Design



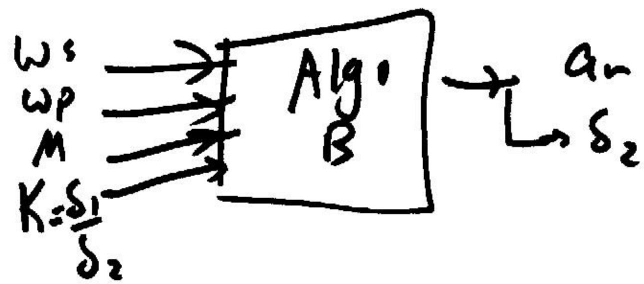


Problem B :

→ Given ω_s, ω_p, M, $K = \frac{\delta_1}{\delta_2}$ →

# of Taps

→ find $a(n)$ to minimize δ_2



Problem C: $E(\omega) = W(\omega)\left[G(\omega) - D(\omega)\right]$

where $W(\omega) =$ positive weighting function $= \begin{cases} \frac{1}{K} & I_1 \\ 1 & I_2 \end{cases}$

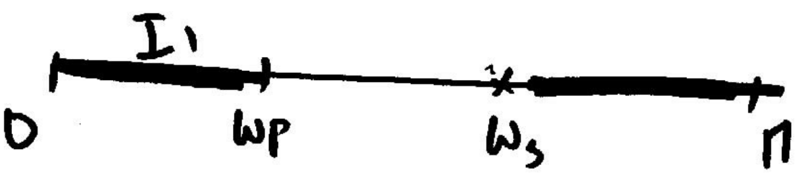
$$G(\omega) = \sum_{n=0}^{M} a(n) \cos(\omega n)$$

$D(\omega) =$ Desired Freq Response $= \begin{cases} 1 & I_1 \\ 0 & I_2 \end{cases}$

→ Find $a(n)$ To minimize $\boxed{\max_{\omega \in F} |E(\omega)|} = \Theta$

when $F = I_1 \cup I_2$

F is a subset of a closed interval $0 \le \omega \le \pi$



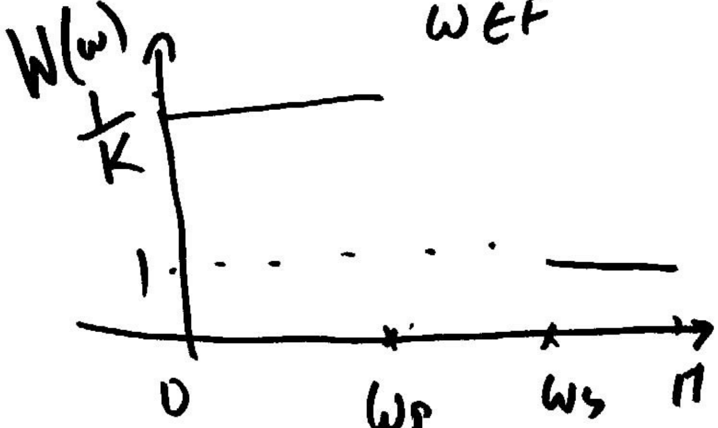
$I_1 =$ passband

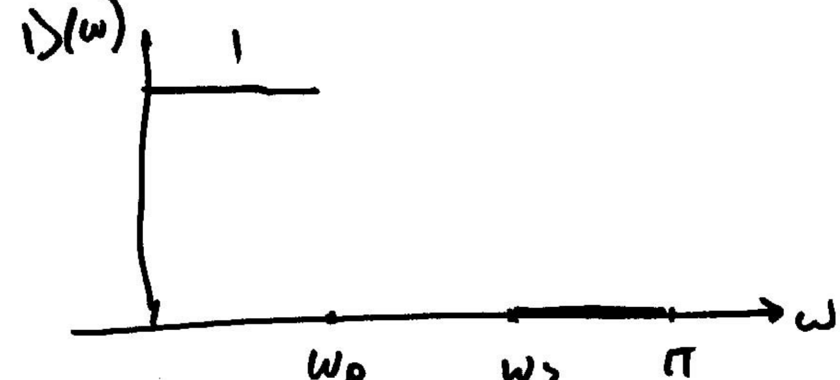
$I_2 =$ stopband

Show: problem C is equivalent to problem B.

start by showing $\theta = \delta_2$

Show $\max\limits_{\omega \in F} |E(\omega)| \overset{??}{=} \delta_2$

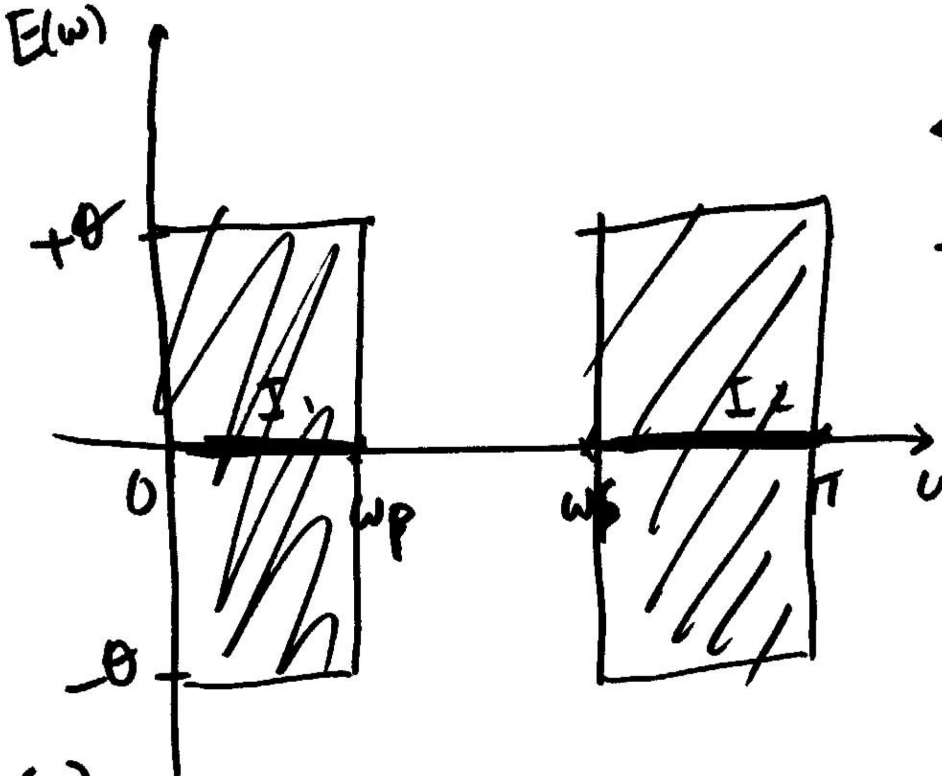


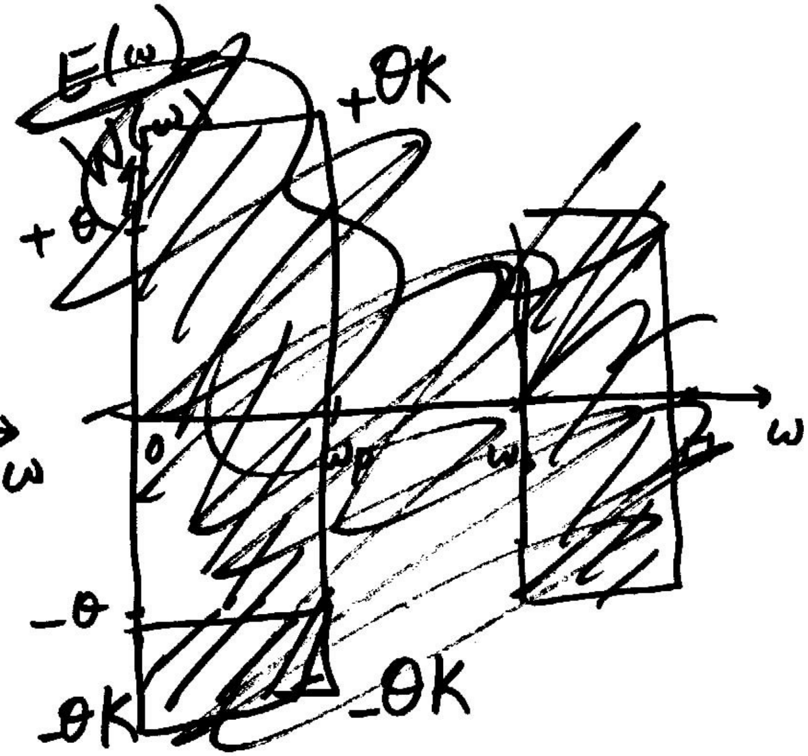


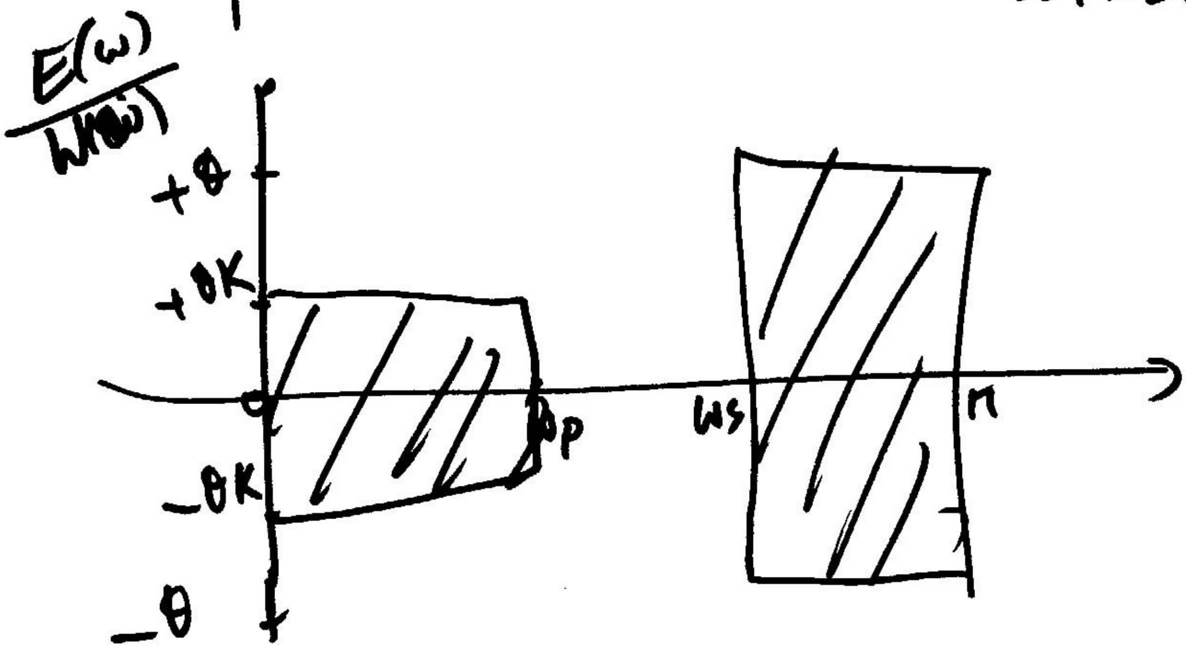
$$E(\omega) = W(\omega)\left[G(\omega) - D(\omega)\right] \Rightarrow \frac{E(\omega)}{W(\omega)} = G(\omega) - D(\omega)$$

$$\Rightarrow G(\omega) = D(\omega) + \frac{E(\omega)}{W(\omega)}$$

By def : $\max\limits_{\omega \in F} |E(\omega)| = \theta$

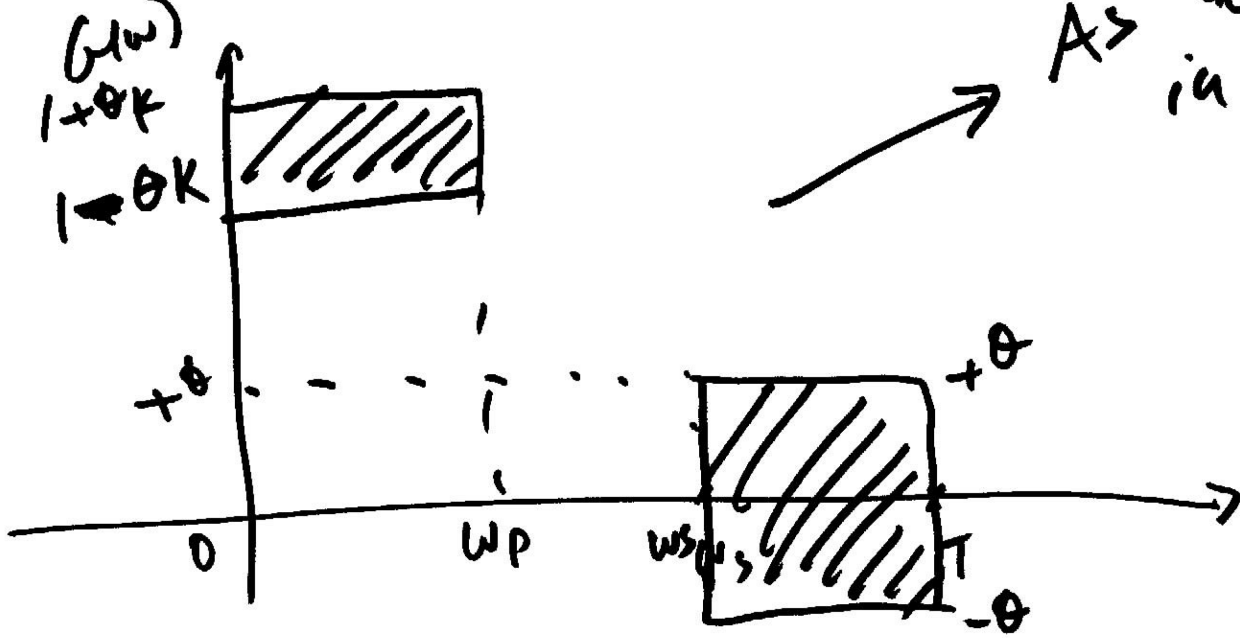






Recall

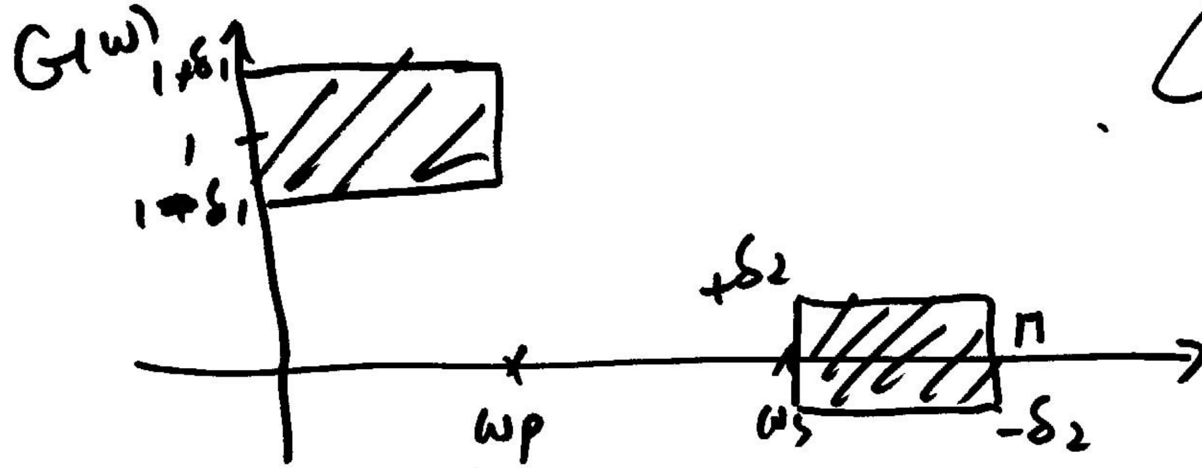
$$G(\omega) = D(\omega) + \frac{E(\omega)}{W(\omega)}$$



As done in Problem C.

$$K = \frac{\delta_1}{\delta_2}$$

How does $G(\omega)$ look like as in Problem A or B?



Conclusion

$$\boxed{\theta = \delta_2}$$

Conclusion:

Problem C: find $a(n)$ To minimize θ

Problem B: " " To minimize δ_2.

$\Longrightarrow$ same problem.

Problem A $\longrightarrow$ Problem B $\longrightarrow$ Problem C.

Solve Problem C:

Alternation Thm:

Assume. If

F = union of closed interval.

Let $P(x)$ be an rth order polynomial.

$$P(x) = \sum_{k=0}^{r} a_k \, x^k$$

D = desired fn continuous in F.

W = positive fns

$$E(x) = W(x) \left[D(x) - P(x) \right]$$

Define $\|E\| = \max_{x \in F} |E(x)|$

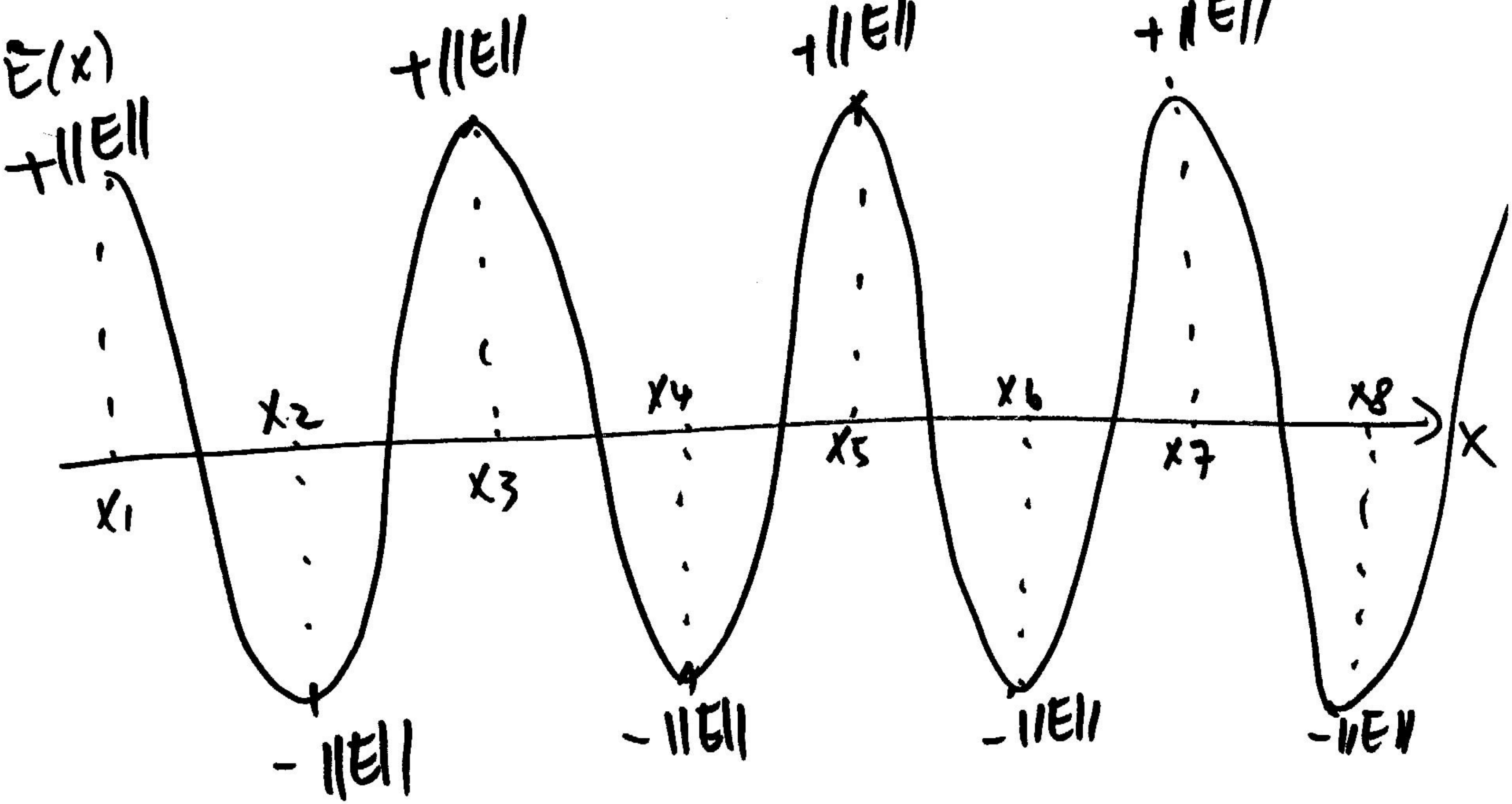
Then.

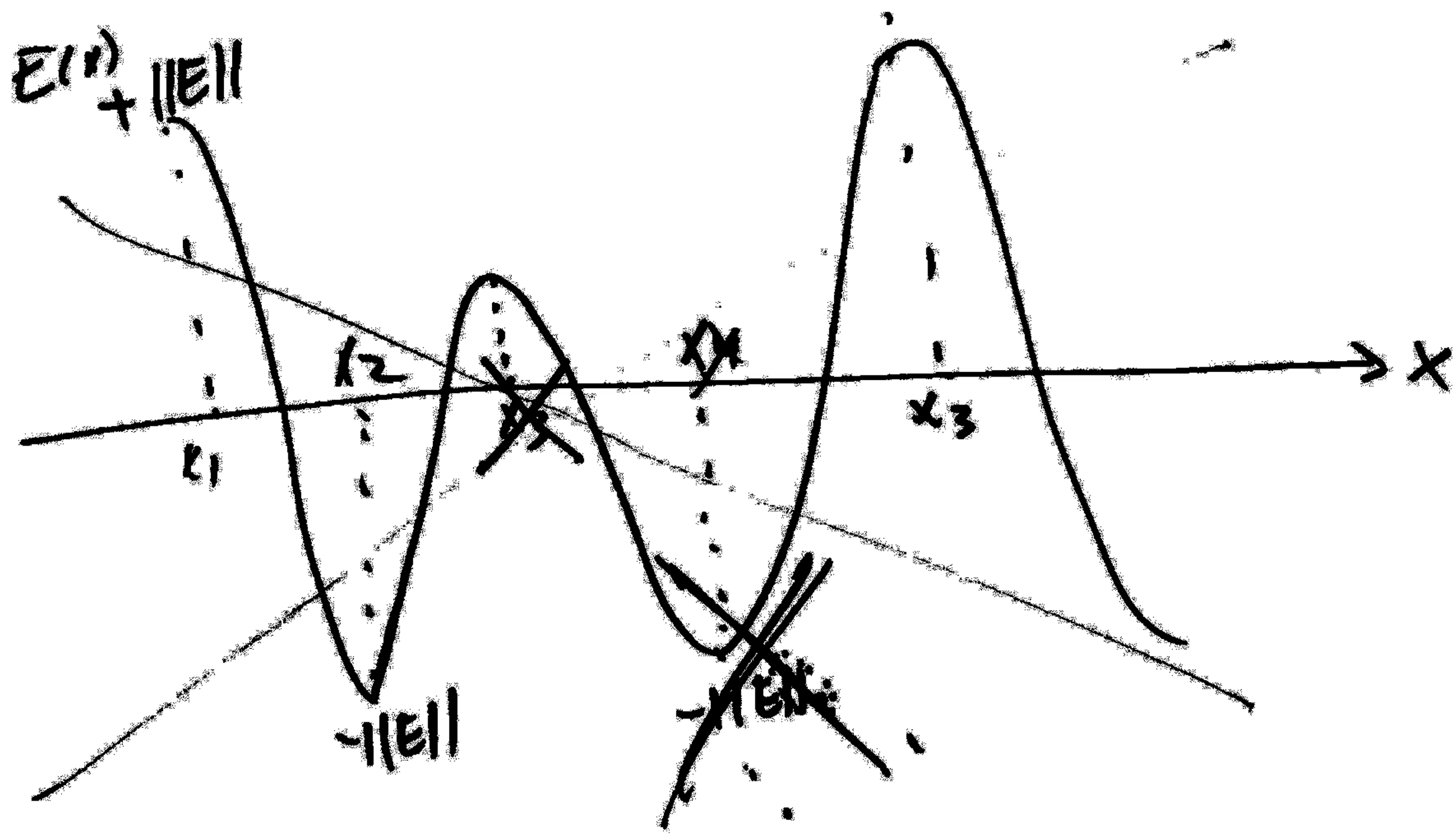
Necessary & Sufficient Condition for $P(x)$ to be a <u>unique</u> rth order polynomial that minimizes $\|E\|$ is that $E(x)$ exhibits <u>at least</u> $r+2$ alternations

i.e. There is <u>at least</u> $r+2$ values of x

$x_1 < x_2 < \cdots < x_{r+2}$ such that

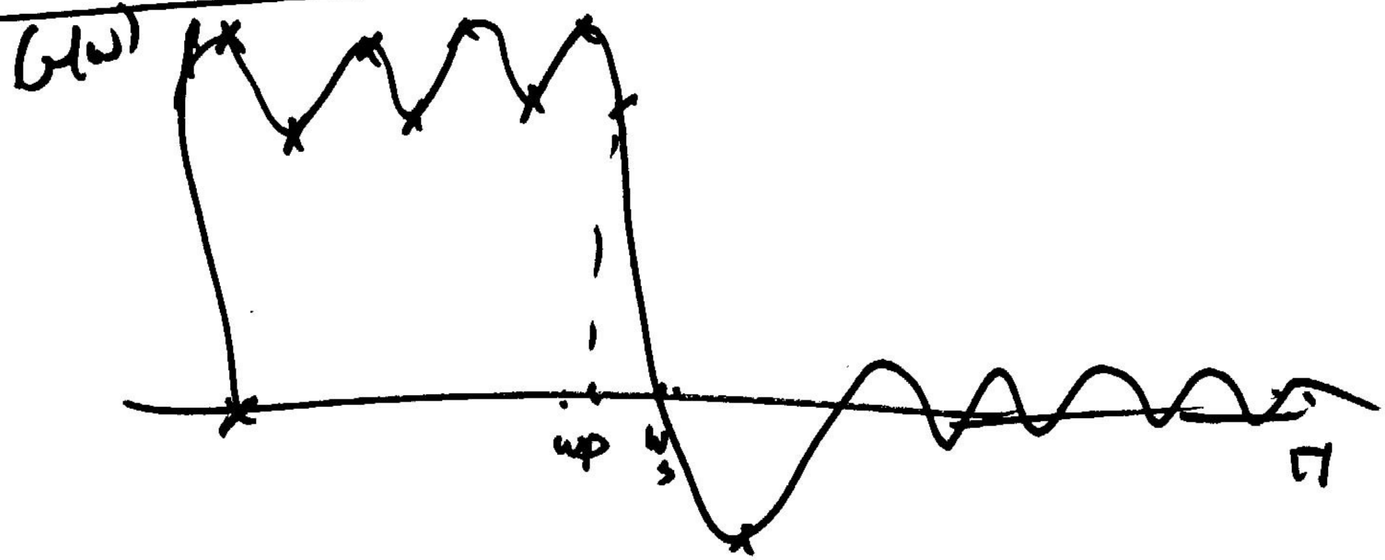
$$E(x_i) = -E(x_{i+1}) = \pm \|E\|$$



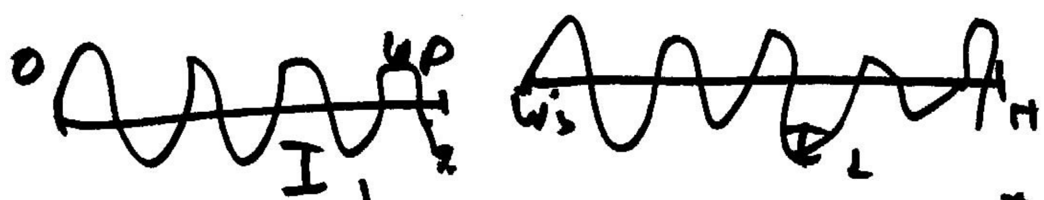


$E^{(n)} + \|E\|$
x_1
x_2
x_4
x_3
x
$-\|E\|$

Recall $G(\omega)$ (last lecture) can have at most $M+1$ local extrema.

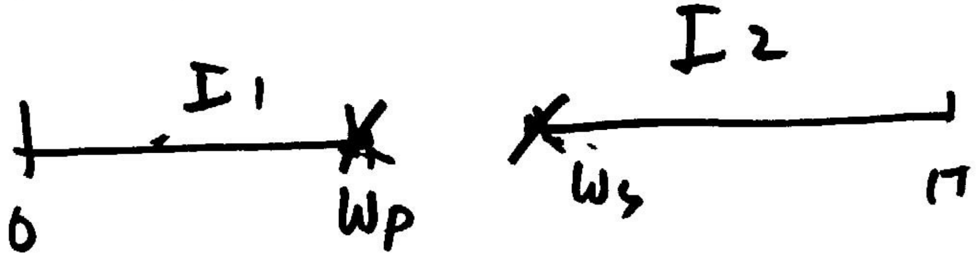


Alt. Thm.



$\downarrow$ cares about $F = I_1 \cup I_2$, possible to have alternation <u>at</u> ω_p & at ω_s even though ω_p, ω_s are not local extrema for G.

$F := I_1 \uplus I_2$



What can we say about ~~Tot #~~ max # of local extrema of $G(\omega)$ in $M+1+2$

$F = I_1 \cup I_2$

$\Rightarrow$ At most $\underline{\underline{M+3}}$ local extrema

for G in $I_1 \cup I_2$

- AH Th at least $r+2$ / $M+2$
- At most $M+3$ extrema

$\Rightarrow$ Either $M+2$ or $M+3$ local extrema in F