Oct 24, 03

Optimal FIR filter Design

- ~~A~~ For low pass filter, alternations always occur at ω_p and ω_s.

  Fig 7.38

- Filter will be equi-ripple except possibly at $\omega=0$ or $\omega=\pi$.

  slope 0

  Fig 7.39.

Alg. for optimal Filter Design

Parks / McClellan.

Given. ω_s, ω_p, $K = \frac{\delta_1}{\delta_2}$, M

find $\underline{\underline{a(n)}}$ to minimize δ_2

Problem B

$G(\omega) = \sum_{n=0}^{M} a(n) \cos \omega n$

$E(\omega_i) = \pm \delta_2 \qquad i = 1, \ldots, M+1$

$E(\omega_i) = -E(\omega_{i+1}) \qquad i = 1, \ldots, M.$

$W(\omega_i)\left[G(\omega_i) - D(\omega_i)\right] = (-1)^{i+1} \delta_2$

ω_i = alternation frequencies.

harder

solve a linear sys of eqns

w_i for $i = 1, \ldots, M+2$

$M+2$ unknowns

$a(n) \longrightarrow M+1$

$\delta_2 \longrightarrow 1$

$M+2$ unknowns

easy

show "hard": show, If I know w_i, how to find $a(n)$ and δ_2.

$$\left.\begin{aligned} G(w_i) &= \sum_{n=0}^{M} a(n) \cos(w_i n) \\ G(w_i) &= \frac{(-1)^{i+1} \delta_2}{W(w_i)} + D(w_i) \end{aligned}\right\} \begin{aligned} &w_i \\ &i = 1, \ldots M+2 \end{aligned}$$

$$\sum_{n=0}^{M} a(n) \cos(w_i n) = \frac{(-1)^{i+1} \delta_2}{W(w_i)} + D(w_i)$$

$i=1$

$$a(0)\cos(w_1 \cdot 0) + a(1)\cos(w_1 \cdot 1) + a(2)\cos(w_1 \cdot 2) + \cdots + a(M)\cos(w_1 \cdot M)$$

$$= \frac{(-1)^{1+1}\,\delta_2}{W(w_1)} + D(w_1)$$

$i=2$

$$a(0)\cos(w_2 \cdot 0) + a(1)\cos(w_2 \cdot 1) \cdots \cdots$$

$$= (\qquad\qquad)$$

$\vdots$

$i = M+2$

$\Rightarrow$ $M+2$ linear equations in $a(n)$

$M+1$ unknown $a(n)$ + δ_2.

$M+2$ unknowns

Solve a linear syst of Eqns. in $M+2$ unknowns.

$$
\underbrace{\begin{array}{c} w_1 \\ w_2 \\ \vdots \\ w_{M+2} \end{array}
\begin{bmatrix}
\cos w_1 0 & \cos w_1 \cdot 1 & \cos w_1 \cdot 2 & \cdots & \cos w_1 M \\
\cos w_2 0 & \cos w_2 \cdot 1 & \cos w_2 2 & \cdots & \cos w_2 M \\
\vdots & & & & \vdots \\
\cos(w_{M+2} \cdot 0) & \cos(w_{M+2} \cdot 1) & \cdots & & \cos w_{M+2} \cdot M
\end{bmatrix}}_{\text{given if } w_i \text{ are known}}
\underbrace{\begin{bmatrix} a(0) \\ a(1) \\ a(2) \\ \vdots \\ a(M) \end{bmatrix}}_{\text{unknown}}
= \underbrace{\delta_2}_{\text{unknown}}
\underbrace{\begin{bmatrix} \frac{(-1)^{1+1}}{W(w_1)} \\ \frac{(-1)^{2+1}}{W(w_2)} \\ \vdots \\ \frac{(-1)^{M+2+1}}{W(w_{M+2})} \end{bmatrix}}_{\text{Known } w_i}
+ \underbrace{\begin{bmatrix} D(w_1) \\ D(w_2) \\ \vdots \\ D(w_{M+2}) \end{bmatrix}}_{\text{Known}}
$$

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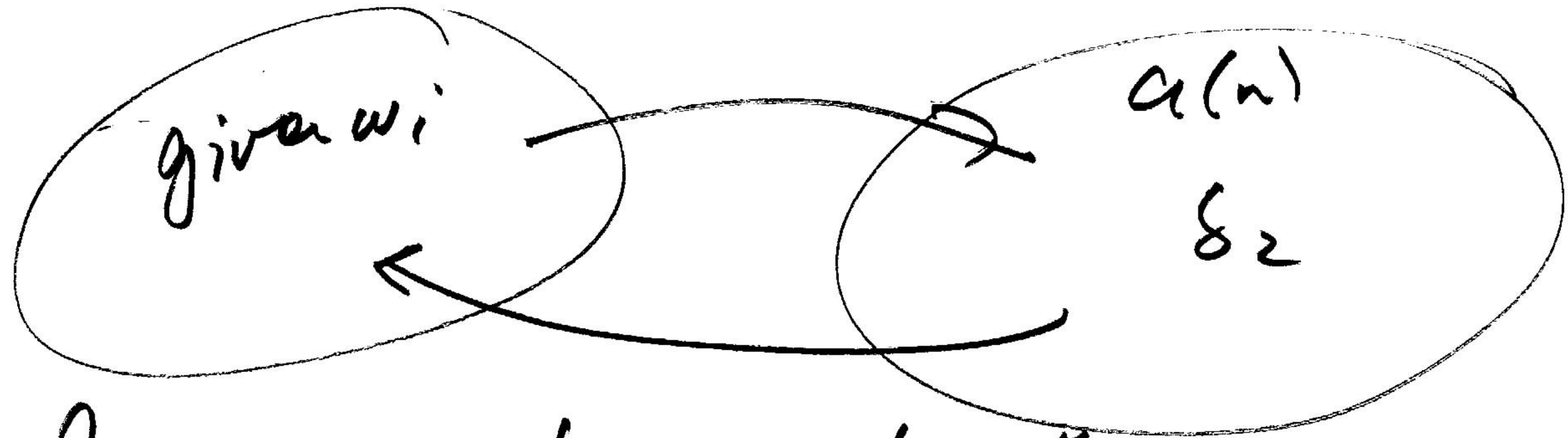

Remez exchange Algorithm.

P & M: showed δ_2 is given by the following expression if ω_i are known:

$$\delta_2 = \frac{\sum_{k=1}^{M+2} b_k \, D(\omega_k)}{\sum_{k=1}^{M+2} \frac{b_k (-1)^{k+1}}{W(\omega_k)}} \qquad \text{where } b_k = \prod_{\substack{i=1 \\ i \neq k}}^{M+2} \frac{1}{\cos\omega_k - \cos\omega_i}$$

Empirical studies

Approx length of filter $\simeq 1 - \dfrac{10\log_{10}(\delta_1\delta_2) - 13}{2.3\ \Delta\omega}$

$\Delta\omega = \omega_s - \omega_p$

Kaiser Window length $\simeq 1 + \dfrac{A - 8}{2.2\,\Delta\omega}$ $\quad A = -20\log_{10}\delta$

Ex $\quad \omega_p = 0.4\pi \quad \omega_s = 0.6\pi$

$\delta_1 = 0.01$
$\delta_2 = 0.001$

optimum filter $\longrightarrow N \simeq 27 = 2M+1$
Kaiser filter $\longrightarrow N \simeq 38$