Oct 29, 03

IIR Filter Design

Restrict T our attention IIR filters with Rational Transfer function.

$$H(z) = \frac{P(z)}{Q(z)}$$ → polynomials in z.

$$P(z) = \sum_{n=0}^{N} \underline{a(n)}\, z^n$$

$$Q(z) = \sum_{n} \underline{b(n)}\, z^n$$

filter Design = Determining coeff.

IIR Filter Design

- Transformation
  - impulse Invariance
  - Bilinear.
- Direct Numerically.

Transformation

1. Given set of Digital Filter Spec. Discrete Time.
2. Transform specs from Discrete Time $\longrightarrow$ continous Time. $z \longrightarrow s$
3. Design continous Time IIR (analog) filter. $H_a(s)$ ← analog.
4. $H_a(s) \xrightarrow{\text{Transform}} H(z)$

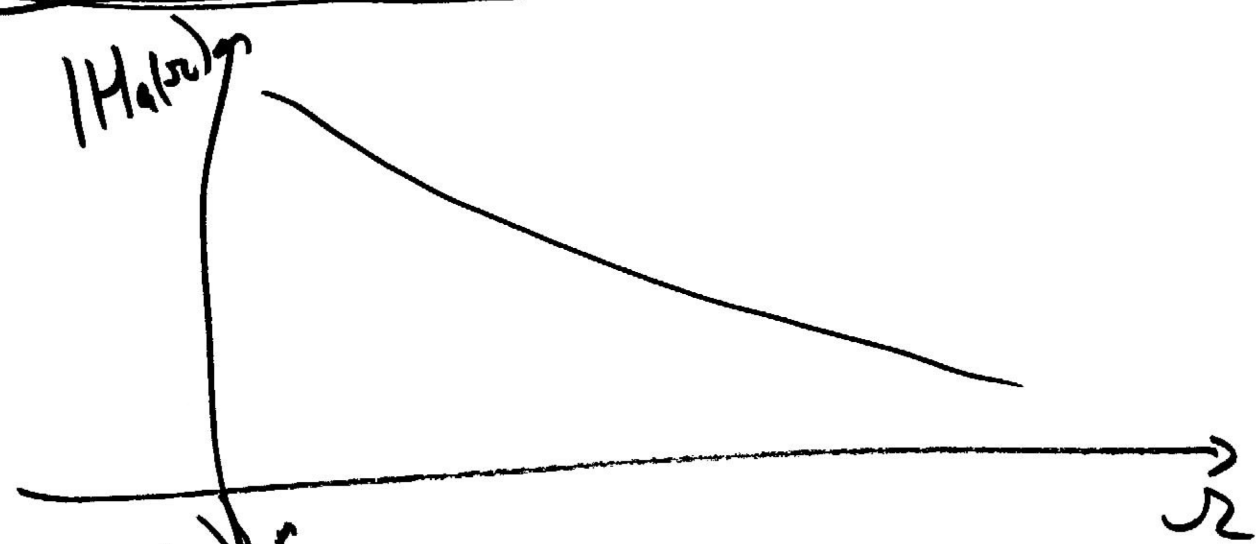
   continous Time $\longrightarrow$ Discrete Time.

   $s \longrightarrow z$.

Continuous Time IIR filters (LPF)

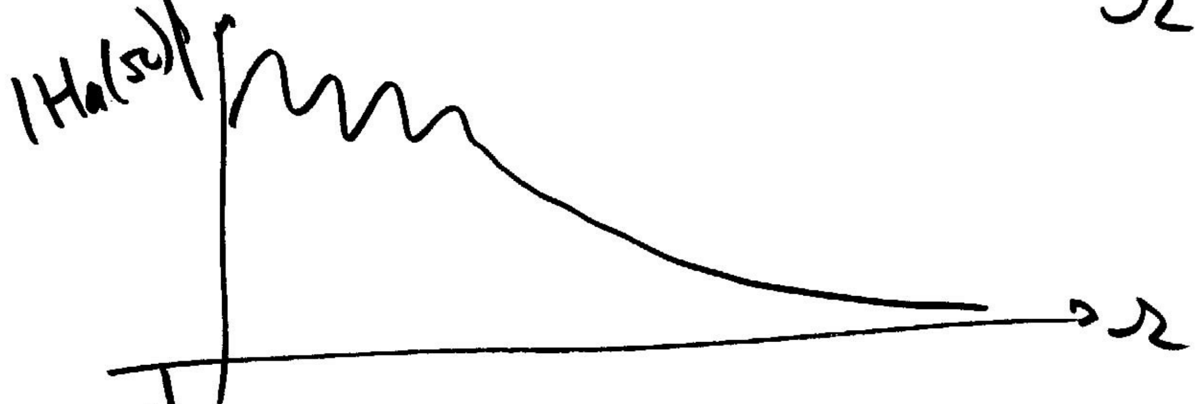
1. Butterworth

monotic in passband + stopband.

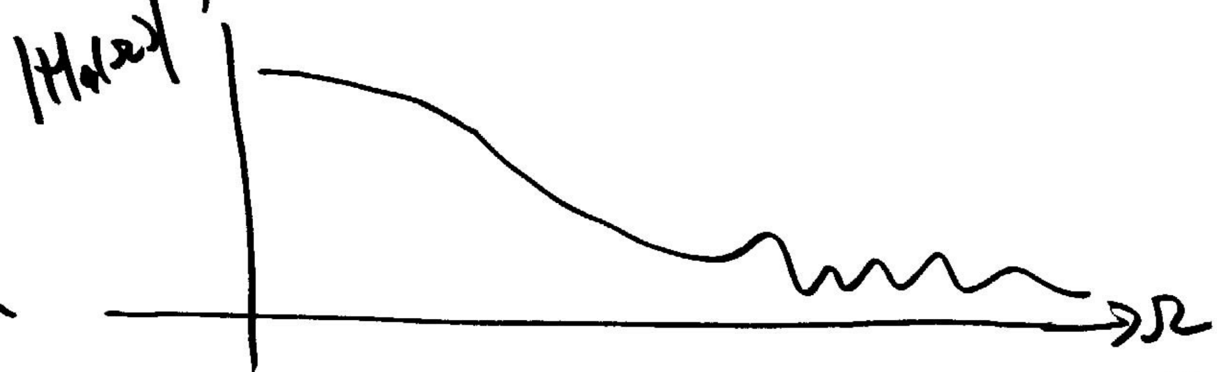


2. Chebychef. filters

- ripple in passband or in stopband but not in both.

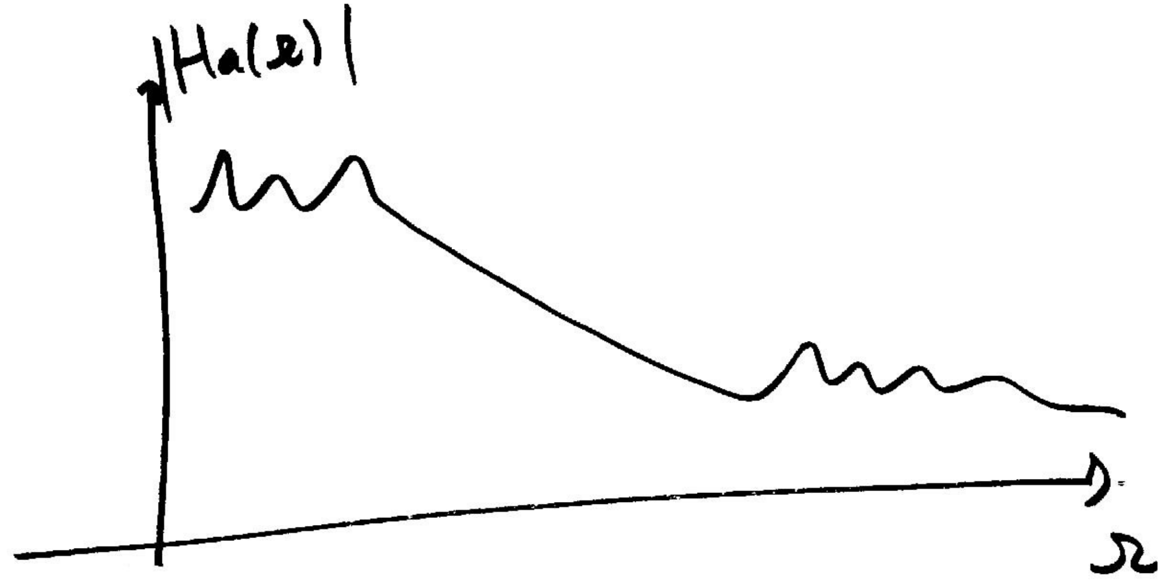


- monotically decreasing in the other



3. Elliptic filters.

- ripple in passband & in stopband



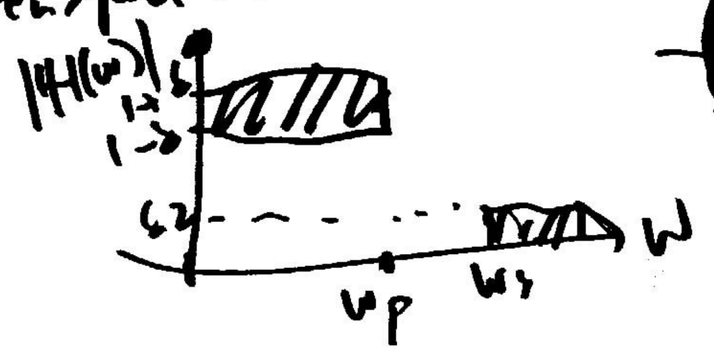
Desirable Properties of Transformation

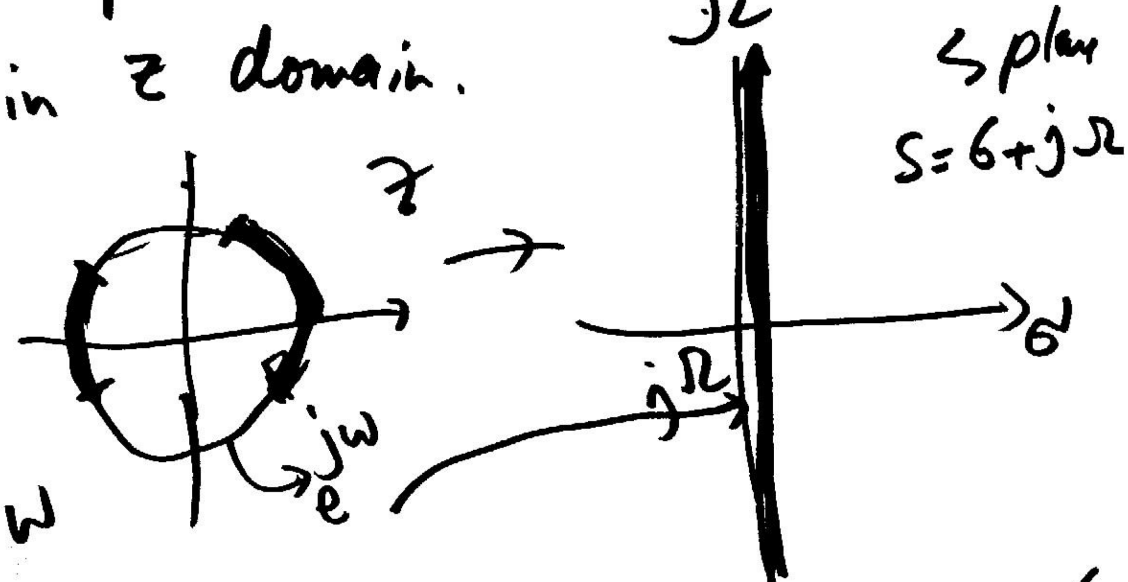
1. Causal stable analog filter to get Tranformed into causal, stable discrete Time filter

$$\underset{\text{causal + stable}}{H_a(s)} \xrightarrow{\text{xform}} \underset{\text{causal + stable.}}{H(z)}$$

2. $j\Omega$ axis in s plane to get Tranfomed into e^{jw} circle in z domain.

Given spec in D.T.





Need this To Translate specs from one domain to another.

$$e^{j\omega} \longleftrightarrow j\Omega$$

z plane. s plane.

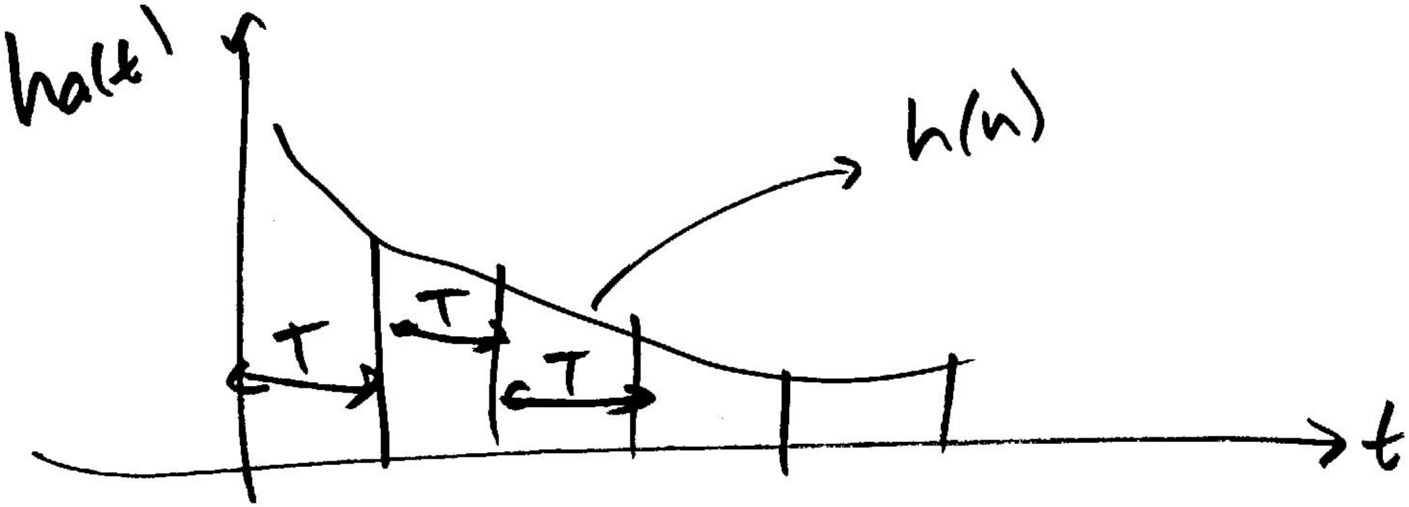
3. Rational $H_a(s)$ analog filter $\longrightarrow$ Rational Discrete Time filter. $H(z)$.

<u>Reason</u>: Diff. Eqns To implement D.T. filter.

$$\cancel{z = \sqrt{s}} \qquad z = \frac{P(s)}{Q(s)}$$

Impulse Invariance Transformation:

$$H_a(s) \rightarrow h_a(t) \longrightarrow h(n) = \left[h_a(t)\right]_{t=nT} \rightarrow H(z)$$



Does This satisfy "Desirable" properties?

Q: Does causal, stable $H_a(s)$ translate into causal & stable $H(z)$?

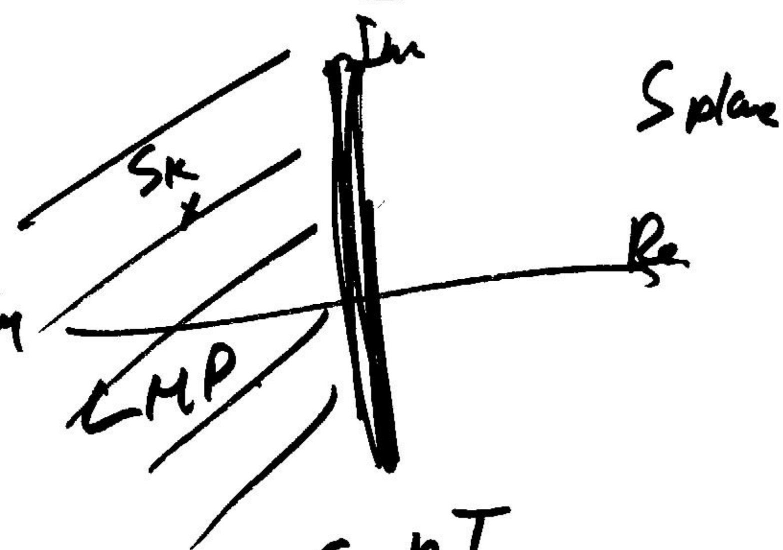
Assume stable + causal. poles are in left half plane.

$$H_a(s) = \sum_k \frac{A_k}{s - s_k}$$

$\mathrm{Re}[s_k] < 0$

$$h_a(t) = \sum_k A_k e^{s_k t} u(t)$$

exploit causality



Transformation

$$h(n) = [h_a(t)]_{t=nT} = \sum_k A_k e^{s_k nT} u(n)$$

causal ✓

$$H(z) = \sum_k A_k \frac{1}{1 - e^{s_k T} z^{-1}}$$

is this stable? is $\left| e^{s_k T} \right| < 1$??

$Re[s_k] < 0 \Rightarrow \left| e^{s_k T} \right| < 1 \Rightarrow$ poles are inside unit circle $\longrightarrow$ stability.

1. showed Causal + stable in Analog $\longrightarrow$
 " " " D.T.

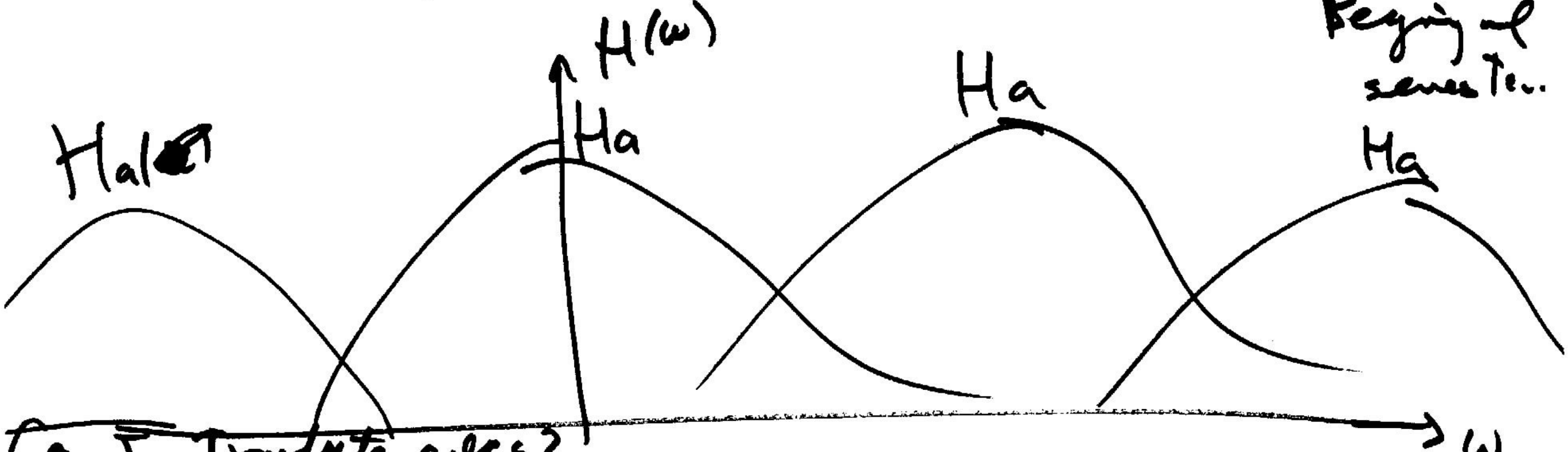
2. showed Rational Xfer fn in Analog $\longrightarrow$
 " " " " D.T.

Q: Does $j\Omega$ in s-plane $\longrightarrow e^{j\omega}$ in z-plane?

$$h(n) = \left[h_a(t)\right]_{t=nT}.$$

$$H(\omega) = \frac{1}{T} \sum_r H_a\left(\frac{\omega}{T} - \frac{2\pi r}{T}\right)$$

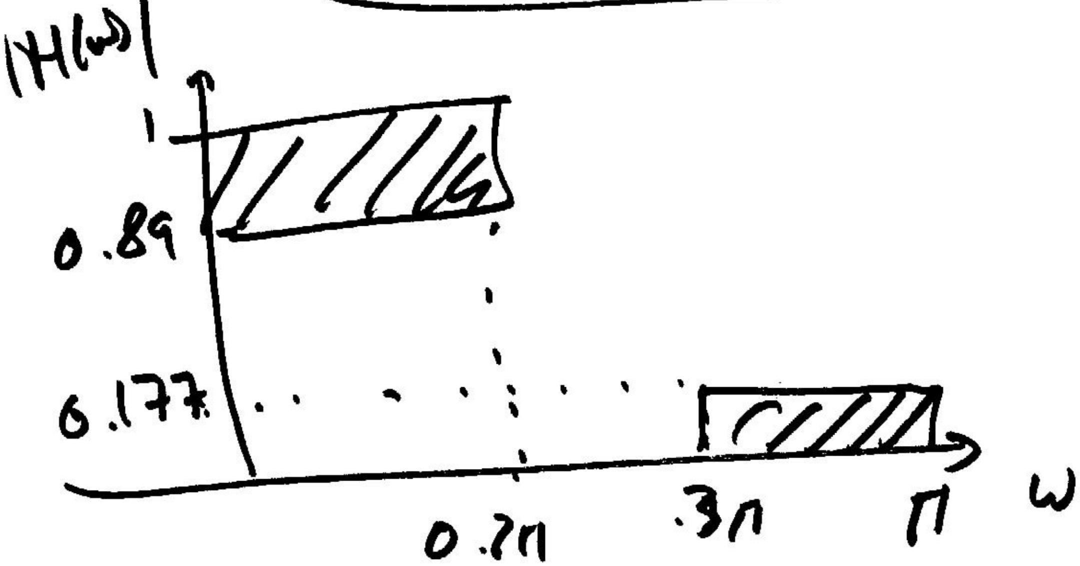
Beginning of series T...



Can I translate specs?

There could be, if there is no aliasing.

Ex Design Using Impulse Invariance.



① Given set of specs in D.T. LPF.

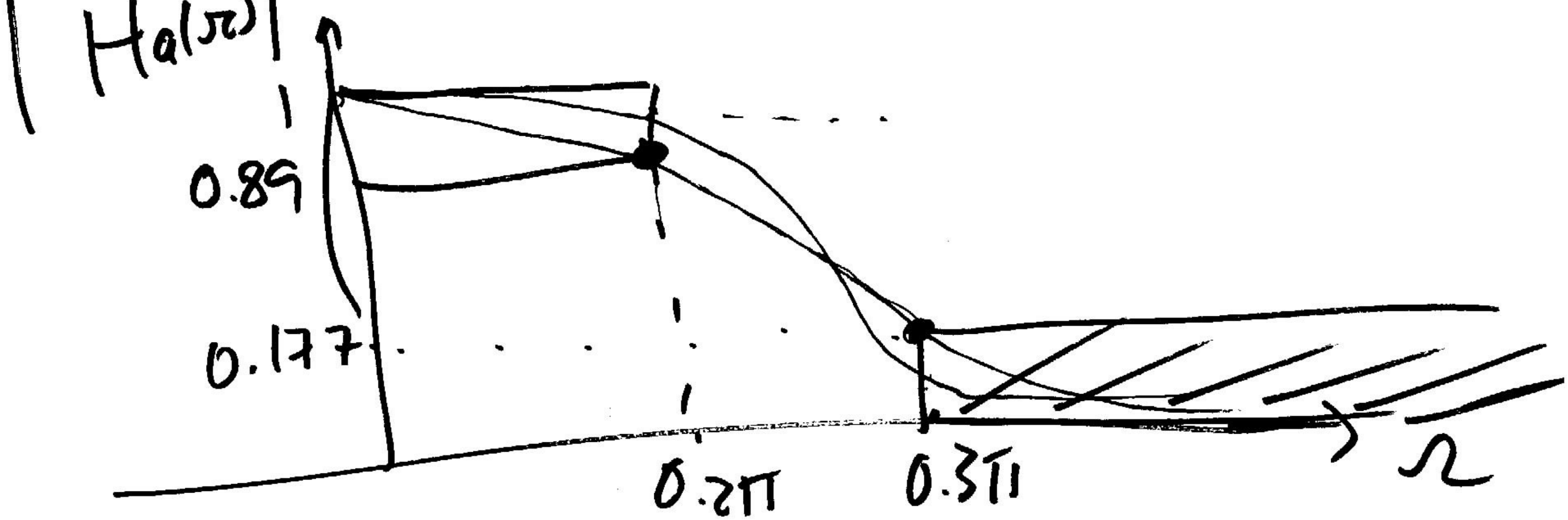
$0.89 \le |H(\omega)| \le 1 \qquad 0 < |\omega| < 0.2\pi$

$|H(\omega)| < 0.177 \qquad 0.3\pi < |\omega| < \pi$

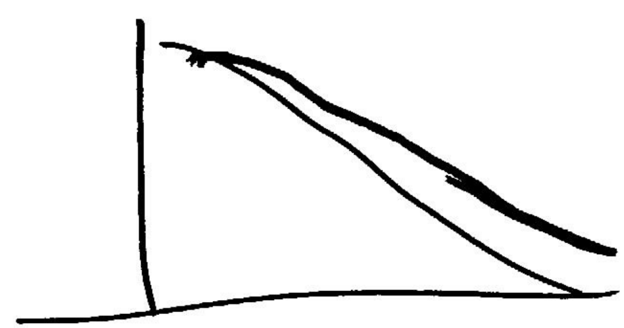
Step ② Translate D.T. specs into C.T. specs.

choose $\underline{T = 1}$

$\omega \rightarrow \Omega$.



③ Design C.T. filter that satisfies spec.



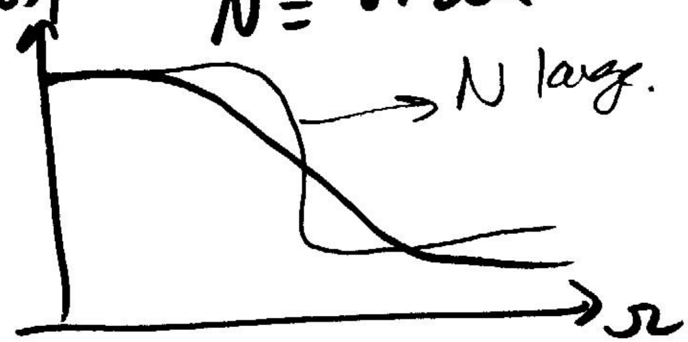
Use Butterworth filter. to minimize order, I will impose.

$$\begin{cases} |H_a(\Omega)|_{\Omega=0.2\pi} = 0.89 \\ |H_a(\Omega)|_{\Omega=0.3\pi} = 0.177 \end{cases}$$

Butterworth 2 min Tutorial on Butterworth of order N.

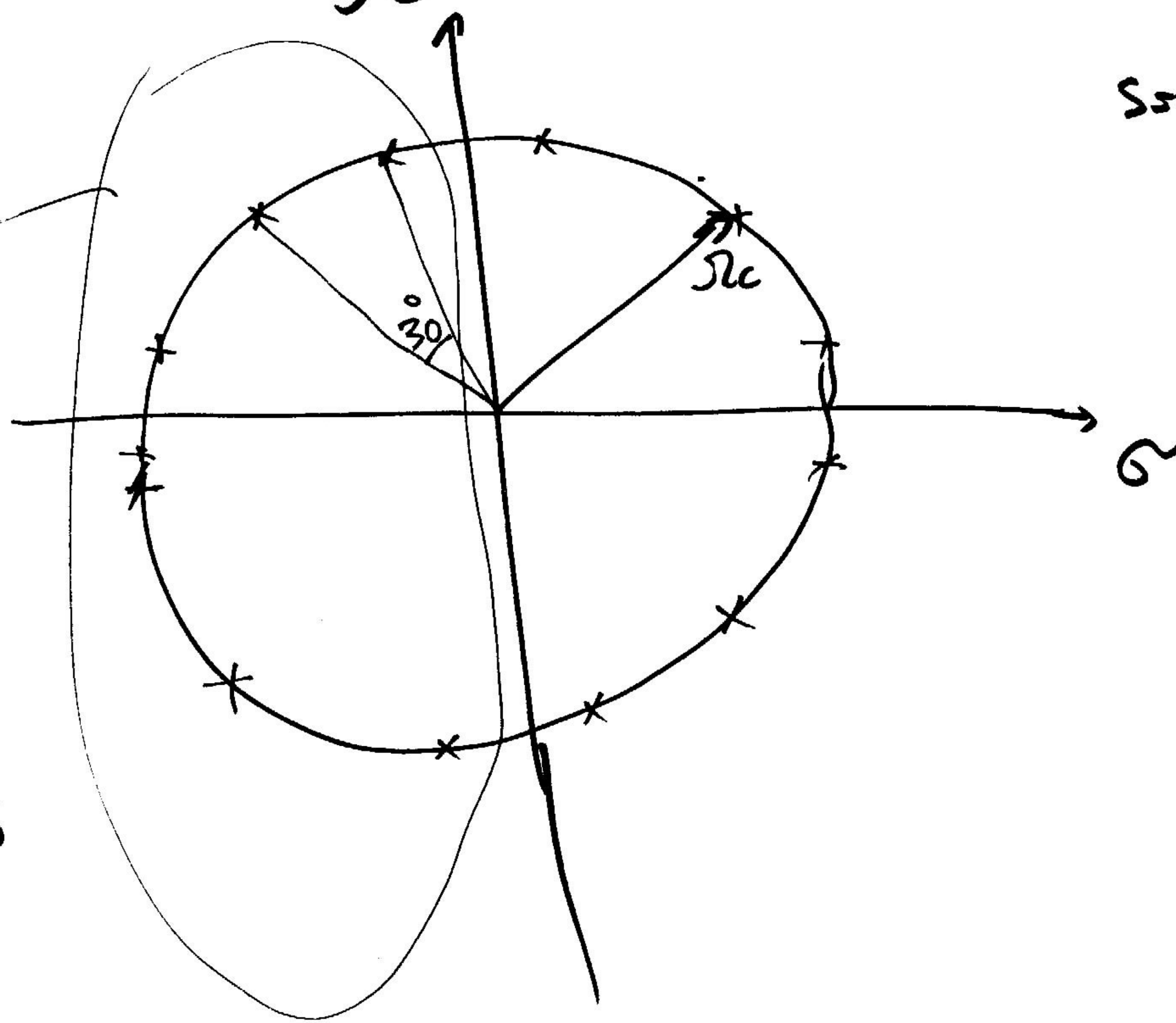
$$|H_a(\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

Ω_c = cut off freq. N = order.



S plane

$s = \sigma + j\Omega$



poles of BW filter

30° deg. apart.

Ex $N = 6$

$$\left|H_a(\Omega)\right|^2_{\Omega=0.2\pi} = (0.89)^2 = \frac{1}{1+\left(\frac{0.2\pi}{\Omega_c}\right)^{2N}} \qquad \text{equ (1)}$$

$$\left|H_a(\Omega)\right|^2_{\Omega=0.3\pi} = (0.177)^2 = \frac{1}{1+\left(\frac{0.3\pi}{\Omega_c}\right)^{2N}} \qquad \text{eqn (2)}$$

2 equns + 2 unknowns.

$$\left.\begin{aligned} N &= 5.88 \\ \Omega_c &= 0.704 \end{aligned}\right\} \rightarrow N = 6$$

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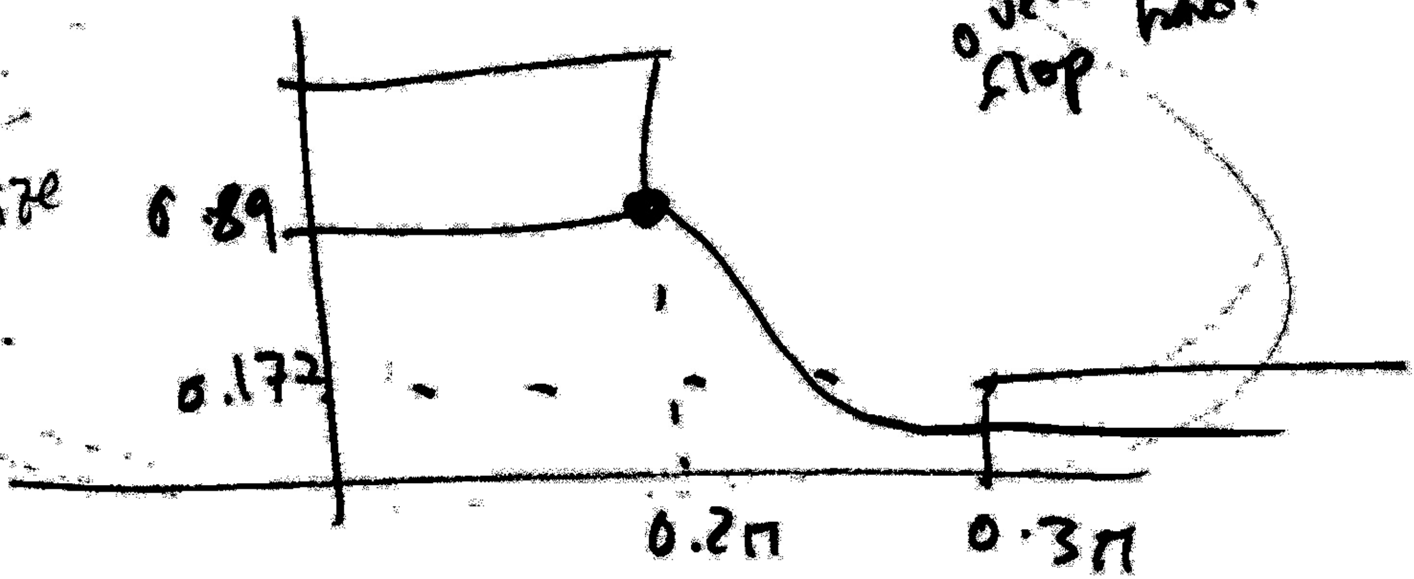
2 choices:

(1) either use equ 1 to compute Ω_c

(2) OR use equ two to compute Ω_c

If I use equ 1

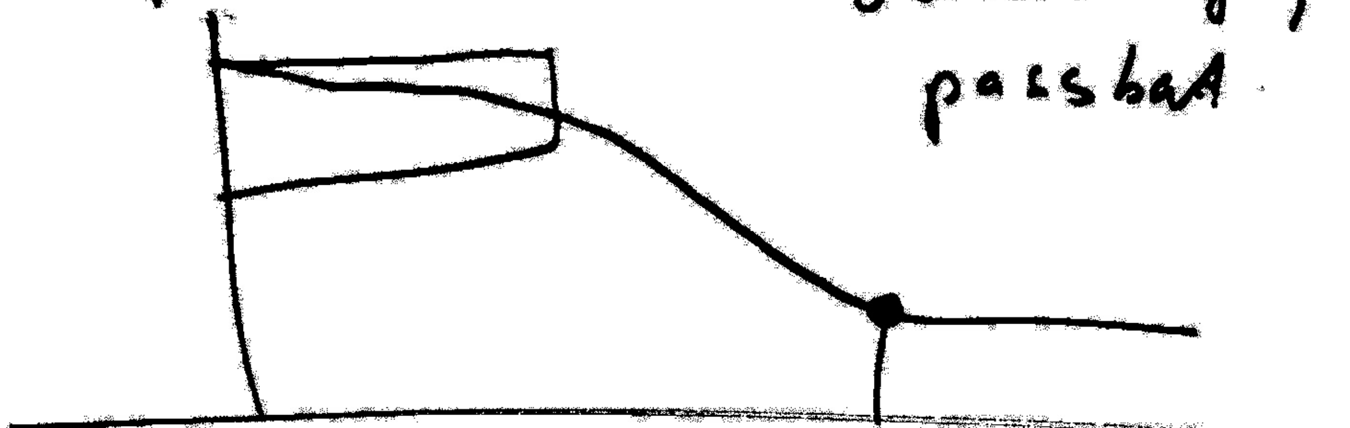
Choose to minimize aliasing



oversatisfying stop band.

If I use eqn 2

oversatisfying passband



Use eqn 1 $\Rightarrow$ $N = 6$

$$(0.89)^2 = \frac{1}{1 + \left(\frac{0.2\pi}{\Omega c}\right)^{12}}$$

$\Rightarrow \Omega c = 0.7032.$

Designed Analog filter.