Oct 31, 03

Bilinear Transformation

$$H_d(z) = \left[H_a(s) \right]_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

* Because it is Rational $s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$

a Rational xfer fn in C.T. $\longrightarrow$ Rational D.T. Transfer fn.

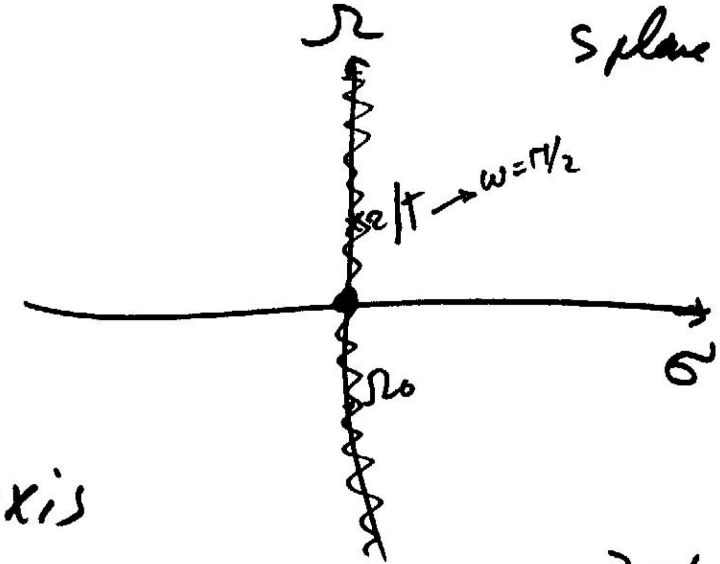
* Does $j\Omega$ axis get mapped onto a circle in $e^{j\omega}$?

Let $z = e^{j\omega} \longrightarrow s = \frac{2}{T}\left(\frac{1-e^{-j\omega}}{1+e^{-j\omega}}\right)$

$$S = \frac{2}{T} \frac{e^{-j\omega/2}\left(e^{j\omega/2} - e^{-j\omega/2}\right)}{e^{-j\omega/2}\left(e^{j\omega/2} + e^{-j\omega/2}\right)}$$

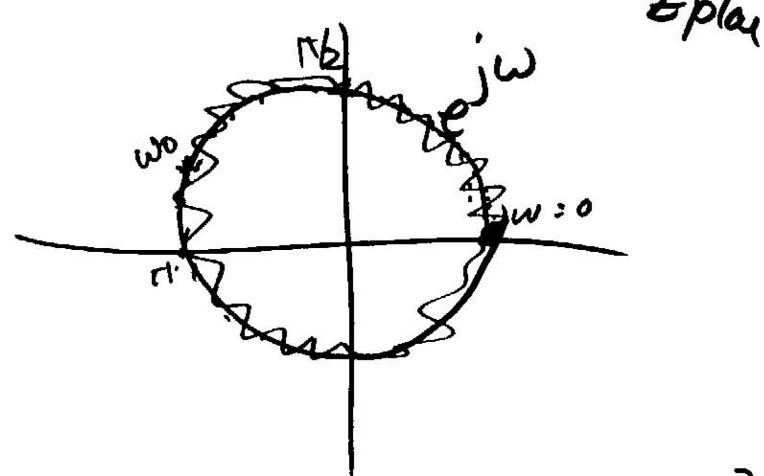
$\Omega_i \leftrightarrow \omega_i$

$$\begin{cases} S = \frac{2}{T} j \tan \frac{\omega}{2} \\ S = \sigma + j\Omega \end{cases}$$



as $\omega : 0 \longrightarrow 2\pi$
S plane. we only Traverse $j\Omega$ axis

$$\Rightarrow \begin{cases} \sigma = 0 \\ \Omega = \frac{2}{T} \tan \frac{\omega}{2} \end{cases}$$



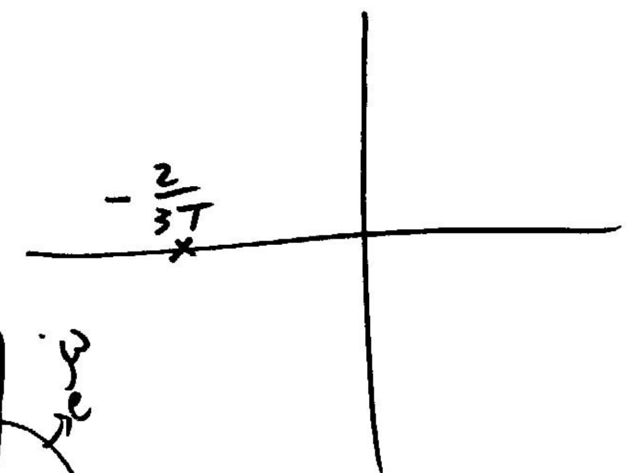
Q Does causal/stable $H_d(s) \longrightarrow$ causal stable $H(z)$?

We need to show LHP in s domain is mapped onto INSIDE $e^{j\omega}$ in z domain & not outside.

- Only need to show it for ONE point.

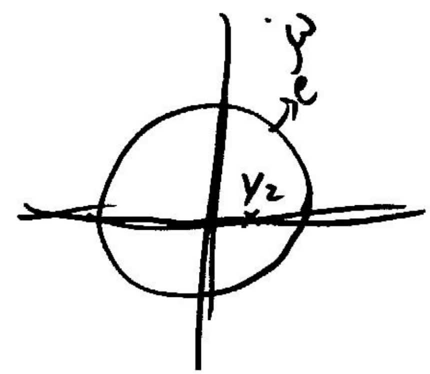
pick a point in s domain in LHP:

$$S = -\frac{2}{3T}$$



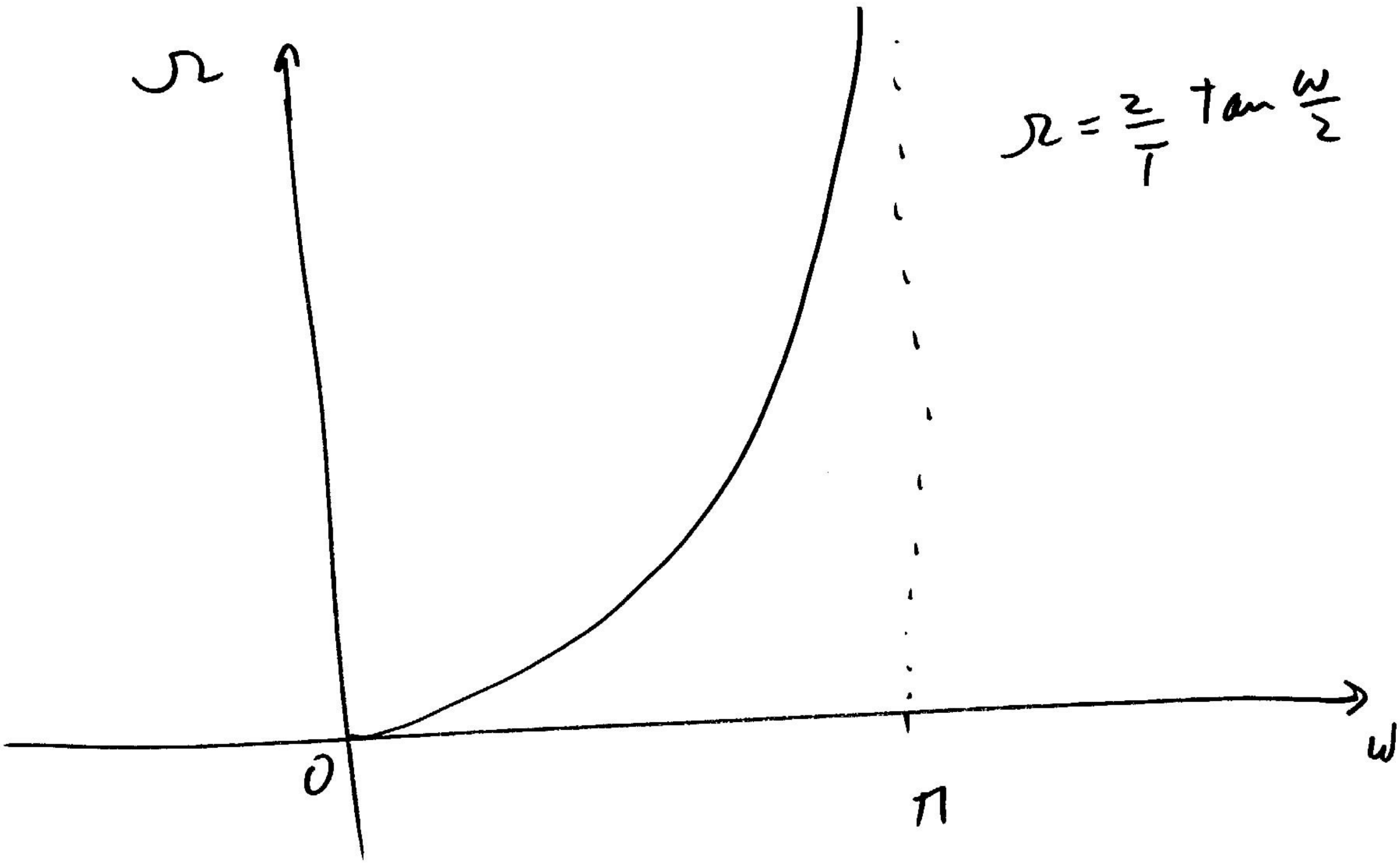
$$S = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = \frac{-2}{3T}$$

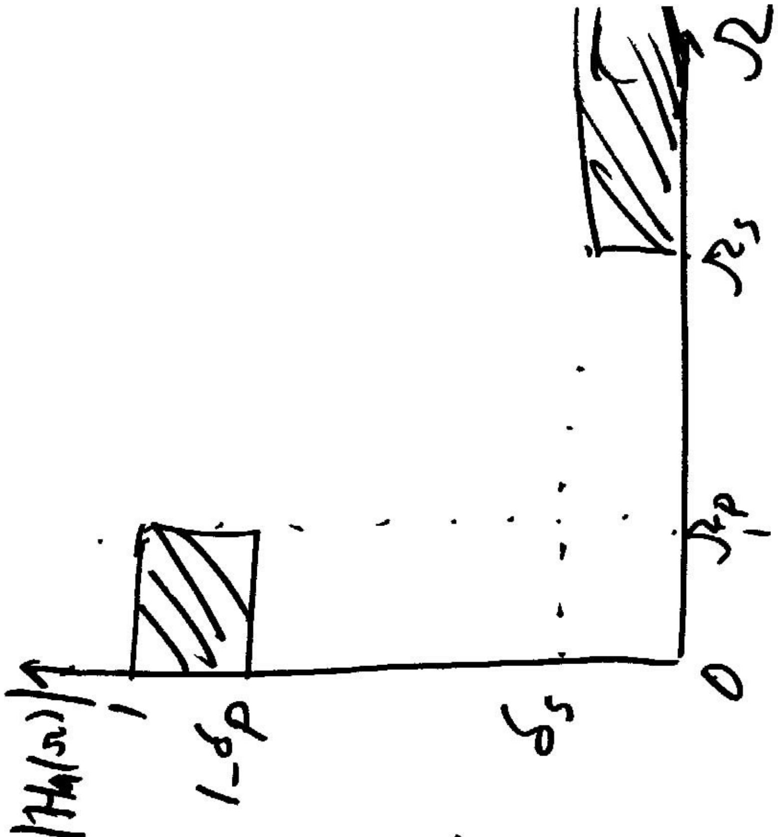
$$\Longrightarrow z = \frac{1}{2} \Rightarrow |z|<1$$

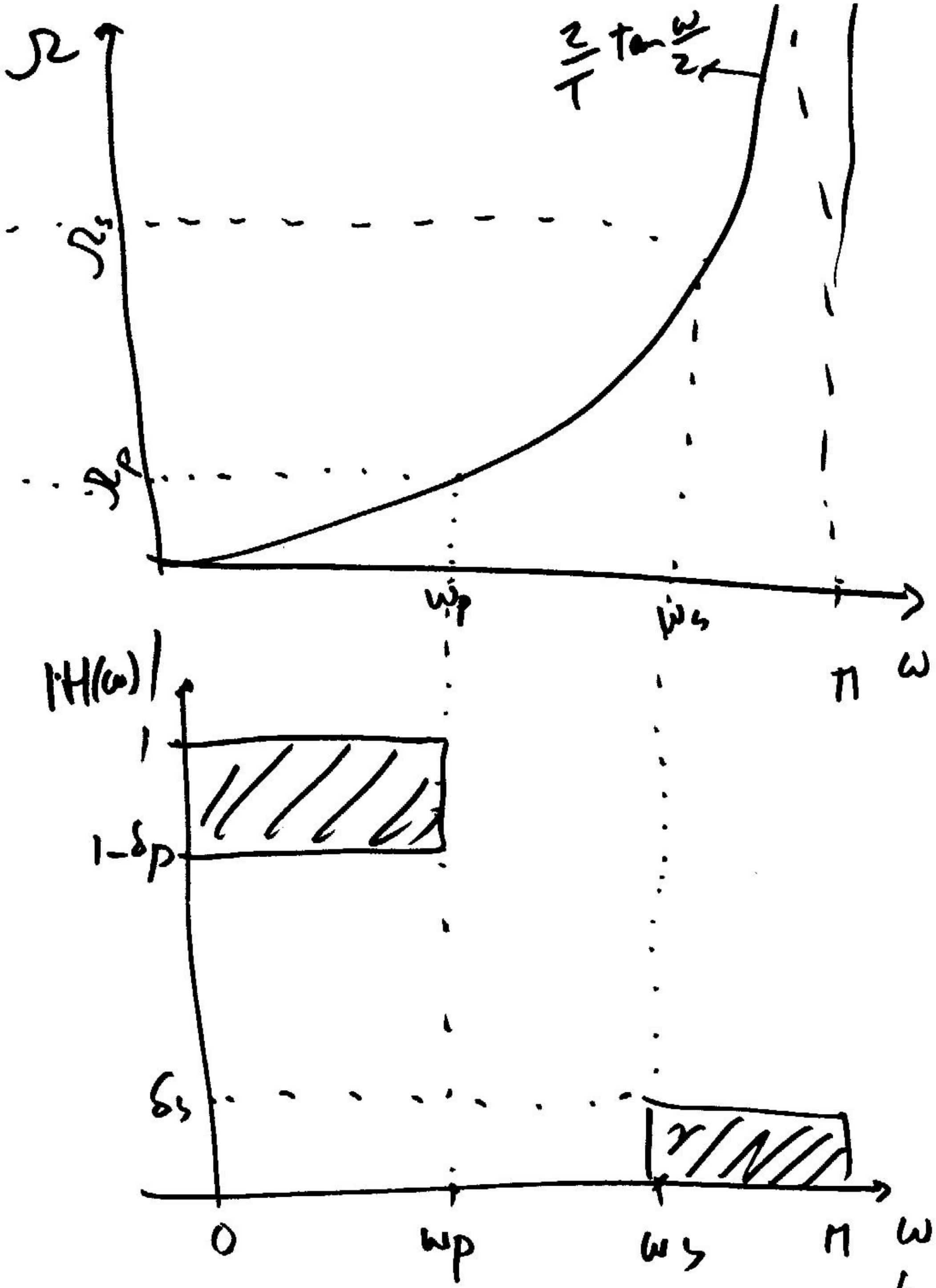


shown causal, stable $H_a(s) \longrightarrow$ causal stable $H_d(z)$

$$\Omega = \frac{2}{T} \tan \frac{\omega}{2}$$

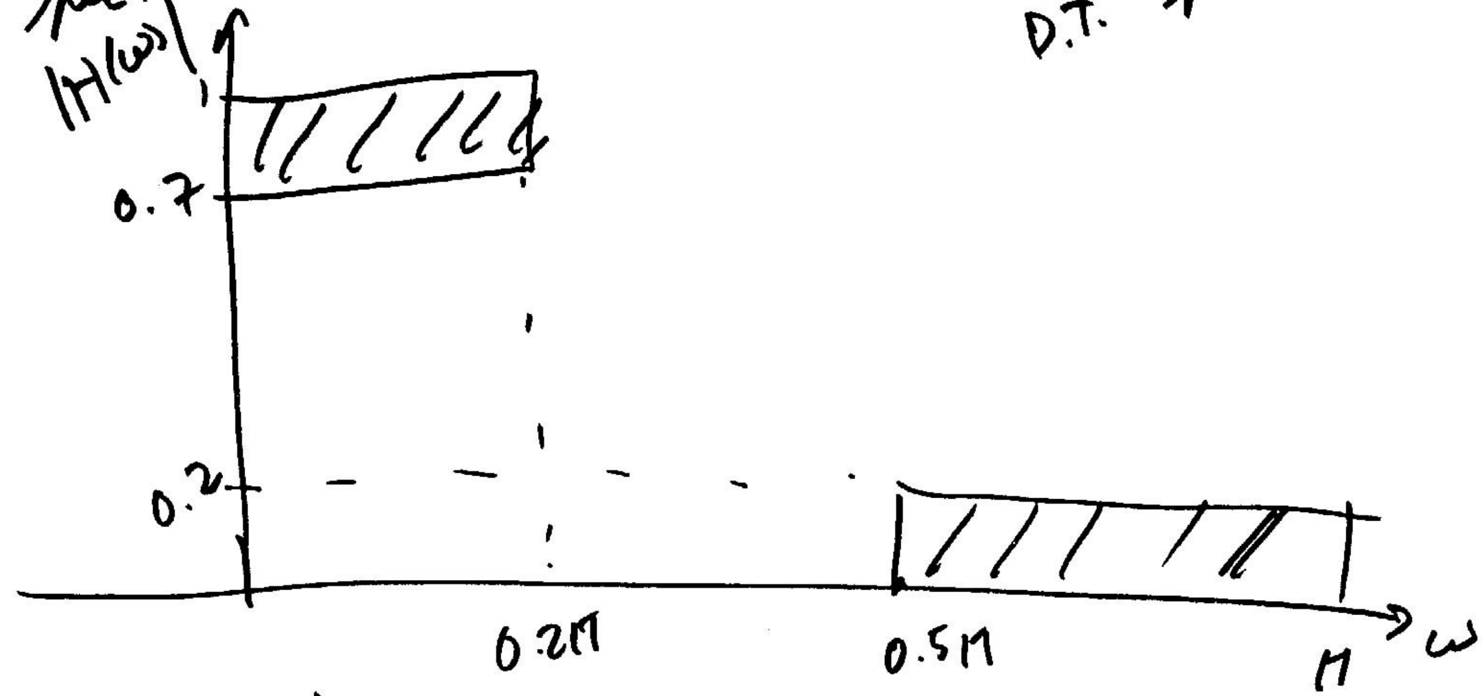






Traslating specs D.T To C.T.

EX given Spec. D.T. spec.



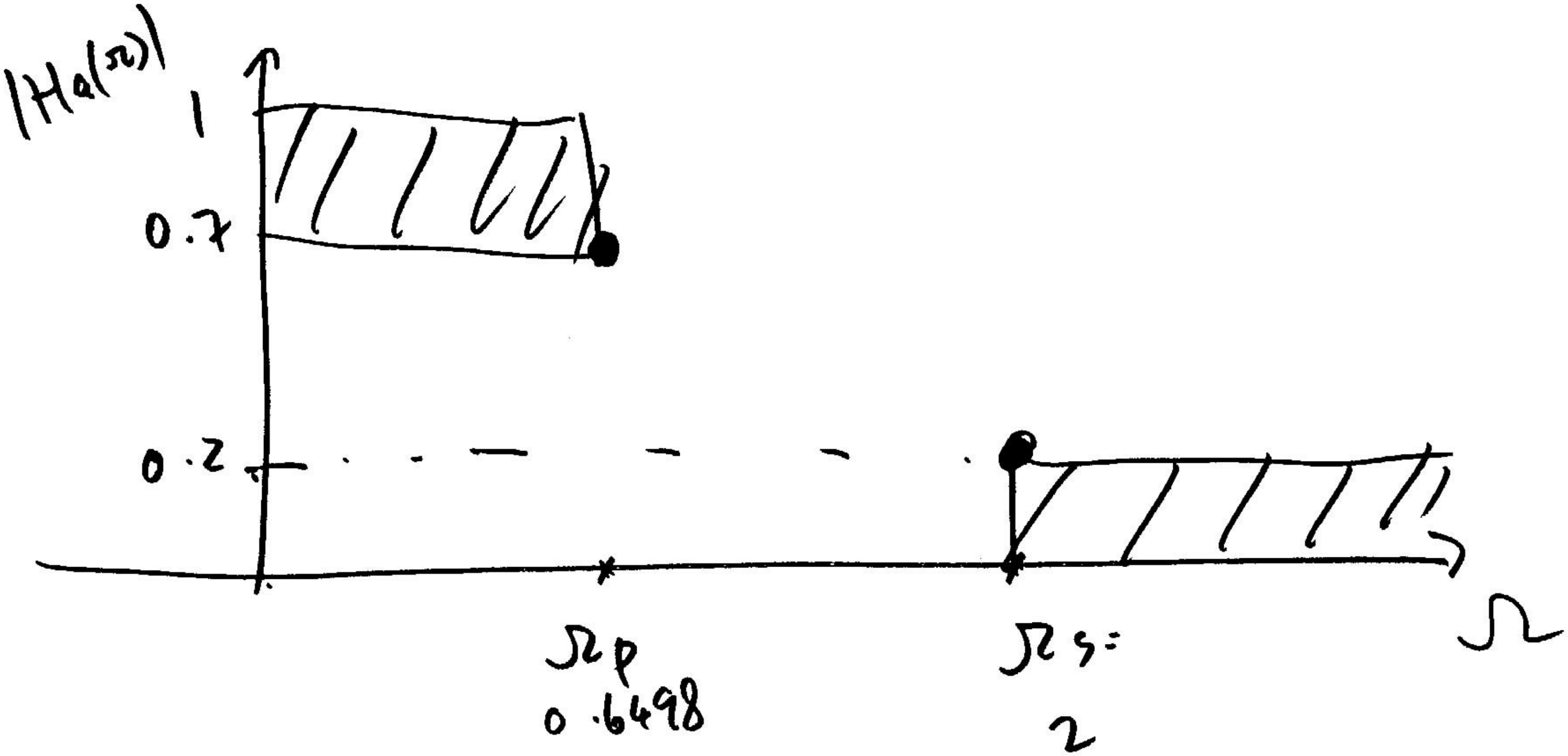
(2) Convert specs D.T. to C.T.

$T=1$

$$\left[\Omega = \frac{2}{T}\tan\frac{\omega}{2}\right]_{T=1} \Longrightarrow \Omega = 2\tan\frac{\omega}{2}$$

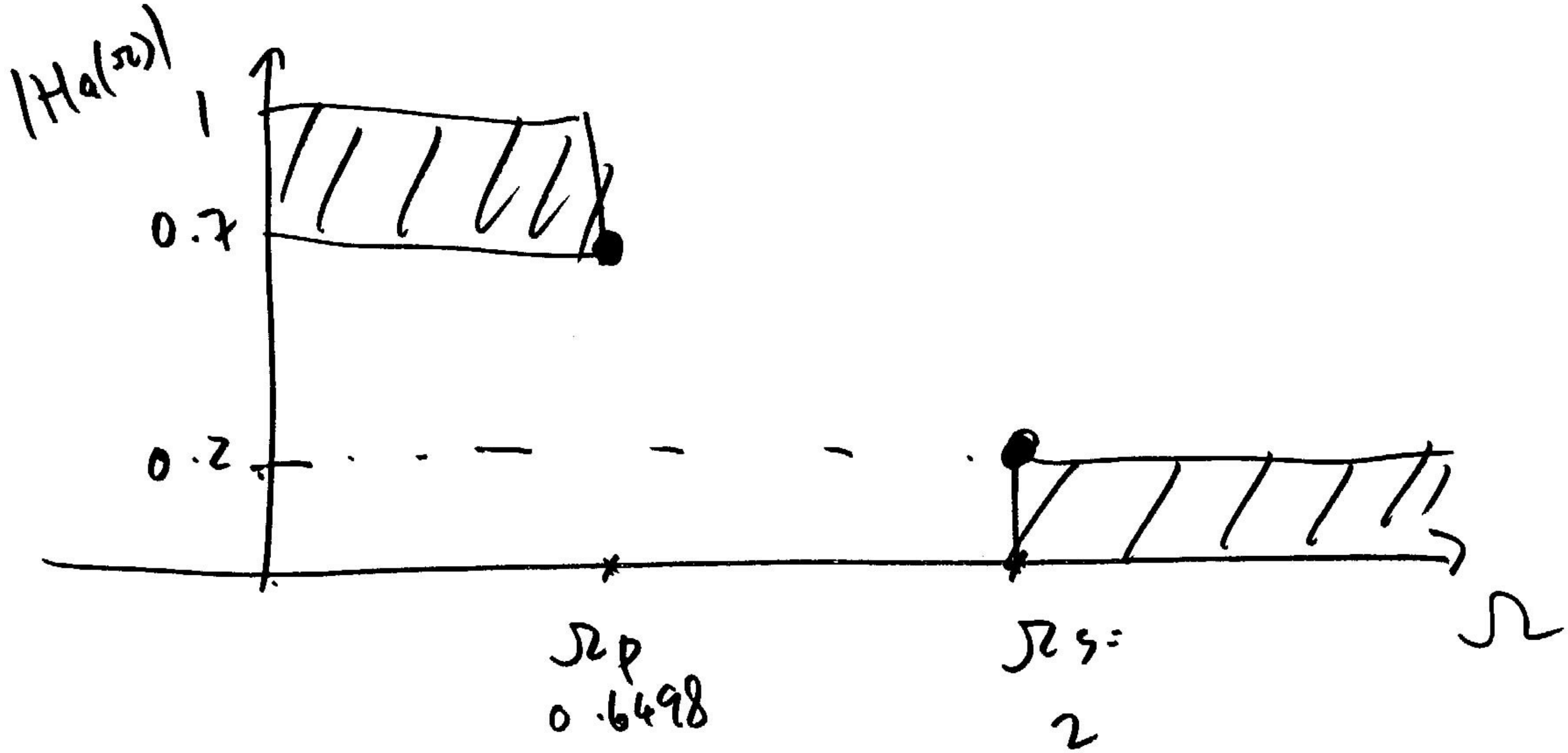
$$\omega_p = 0.2\pi \Longrightarrow \Omega_p = 2\tan\frac{0.2\pi}{2} = 0.6498$$

$$\omega_s = 0.5\pi \Longrightarrow \Omega_s = 2\tan\frac{0.5\pi}{2} = 2$$



(3) Design IIR analog filter.

$$|H_a(\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$



(3) Design IIR analog filter.

$$|H_a(\Omega)|^2 = \frac{1}{1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

pass through Ω_p:

$$(0.7)^2 = \frac{1}{1+\left(\frac{0.6498}{\Omega_c}\right)^{2N}}$$

pass through Ω_s:

$$(0.2)^2 = \frac{1}{1+\left(\frac{2}{\Omega_c}\right)^{2N}} \leftarrow \text{eqn 2}$$

$\Longrightarrow$

$N = 1.3957$

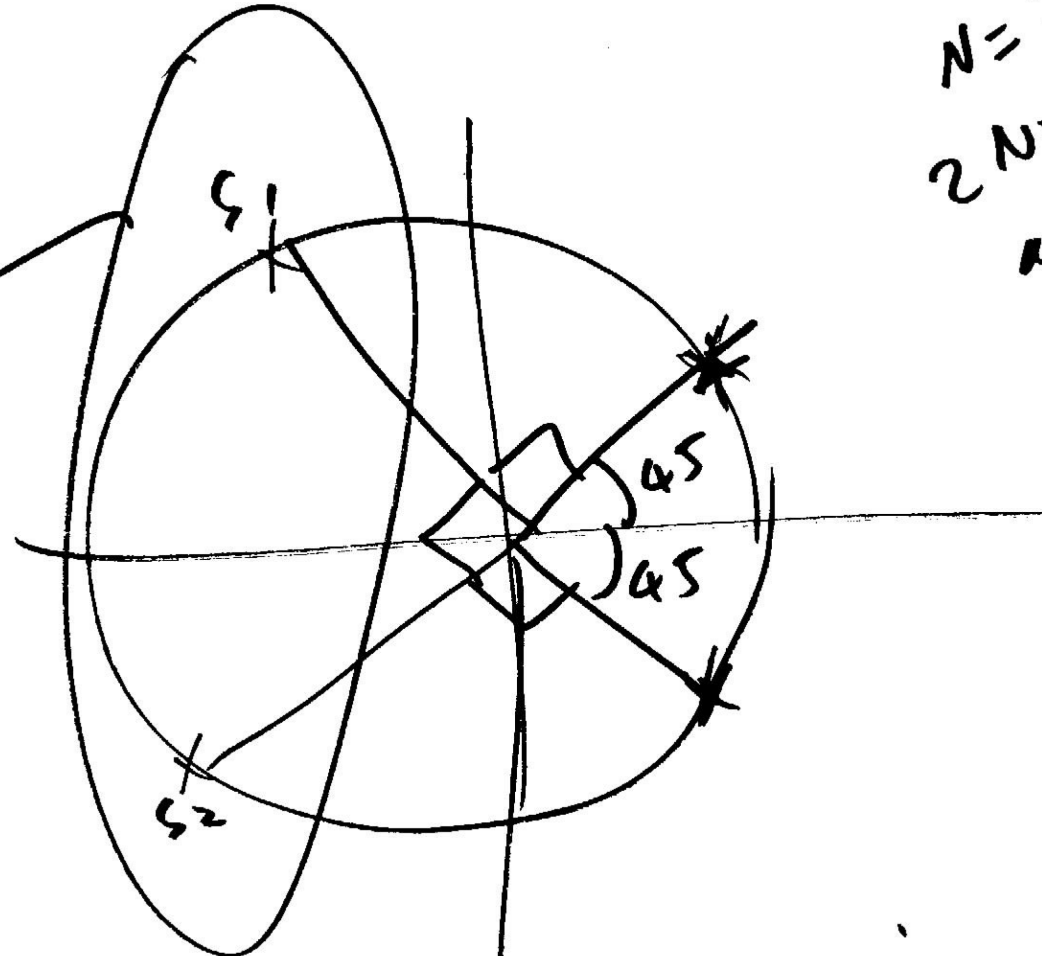
$\Omega_c = 0.6405$

$\Longrightarrow N = 2$

which eqn to use Ω_c $\Rightarrow$ makes no differen.

eqn 2 $\Longrightarrow$ $\Omega_c = 0.6437$

poles for butterwal.



$N = 2$

$2N = 4$

angle $= \frac{360}{4} = 90^\circ$

$s_1 = -0.4549 + 0.4549\,j$

$s_2 = -0.4549 - 0.4549\,j$

$$H_a(s) = \frac{(0.6437)^2}{(s - s_1)(s - s_2)}$$

④ $$s = \frac{2}{1}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$$

$T = 1$

$$H(z) = \frac{0.4139 + 0.829 z^{-1} + 0.4139 z^{-2}}{6.23 - 7.1 z^{-1} + 2.59 z^{-2}}$$