Nov 19, 03

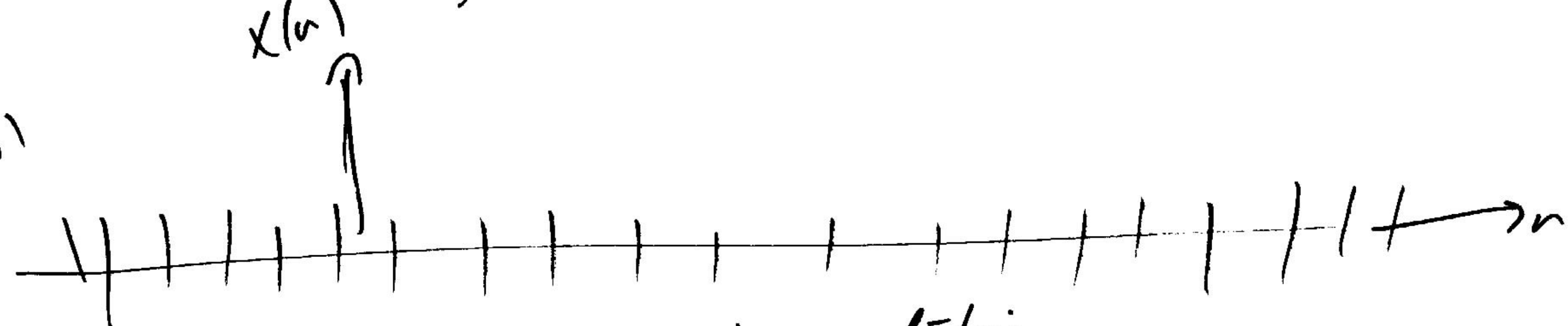
Using DFT for filtering vly long Sequences

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

$x(n)$: very long.

$h(n)$: FIR. small # of Taps. P Taps.

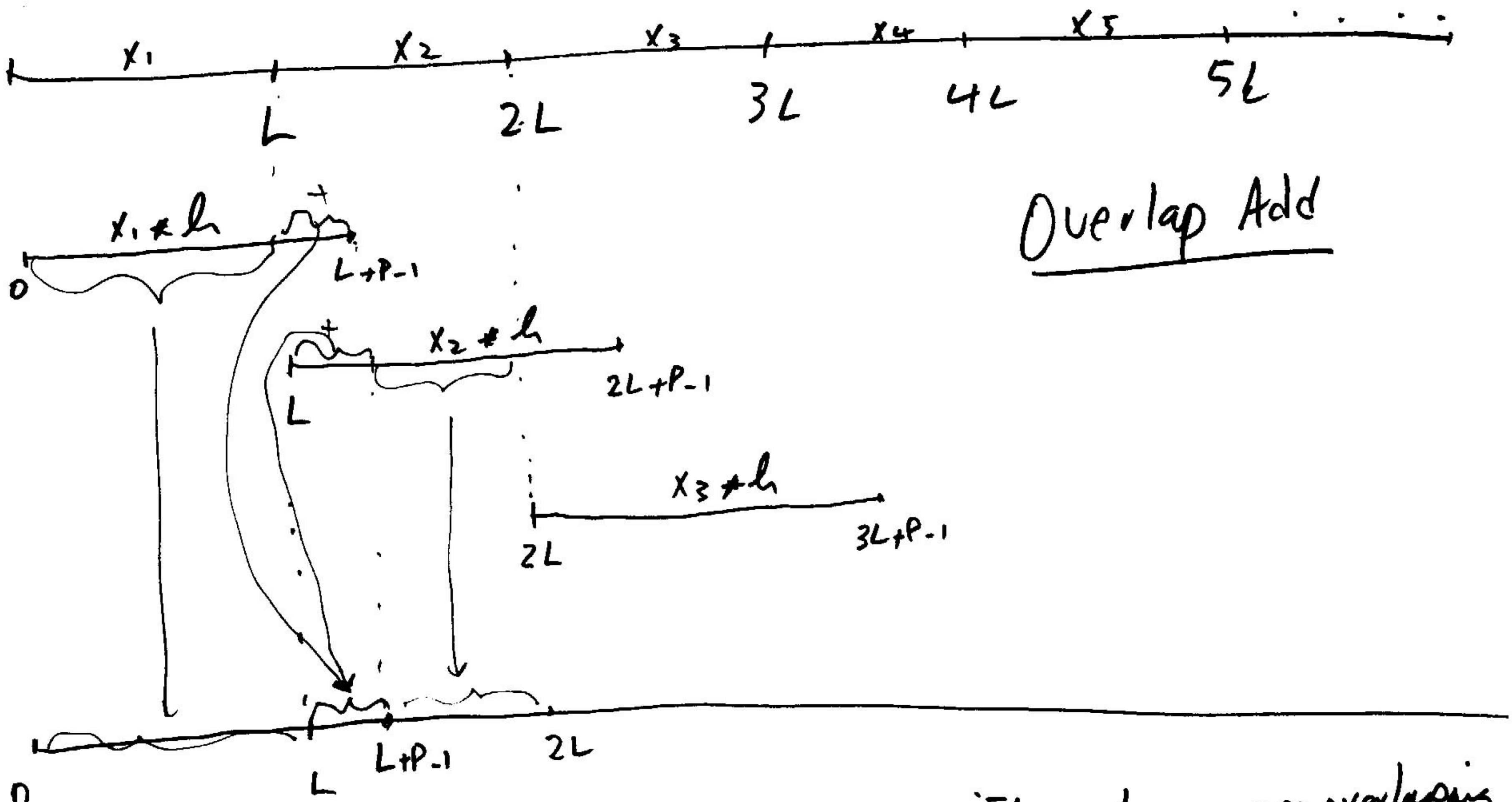
x(n)



Exploit linearity property of convolution

$$[x_1(n) + x_2(n)] * h = (x_1 * h) + (x_2 * h)$$

Overlap Add



① * segment ∞ly long sequence into L, nonoverlapping chuncks.

(2) Convolve each chunk with h
$(L+P-1)$ new point.

(3) add up all the convolutions.

Overlap Save . $L >> P$.

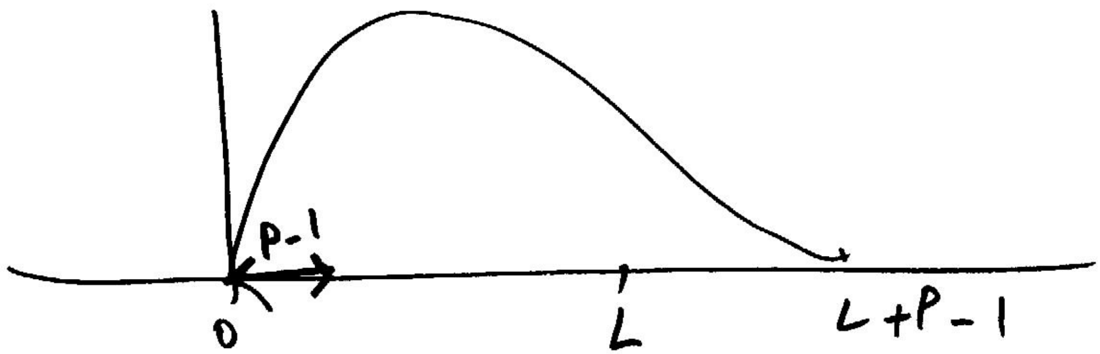
Thought experiment

$$x_1 \longrightarrow L \text{ pt seq.}, \quad x_2 \longrightarrow P \text{ pt seq.} \quad \Big\} \quad x_1 * x_2 \longrightarrow L+P-1 \text{ pt seq.}$$

- correct :
  - pad with zeros to get $L+P-1$ pt sequence
  - multiply 2 , $L+P-1$ point DFTS.
  - Take $L+P-1$ point IDFT.

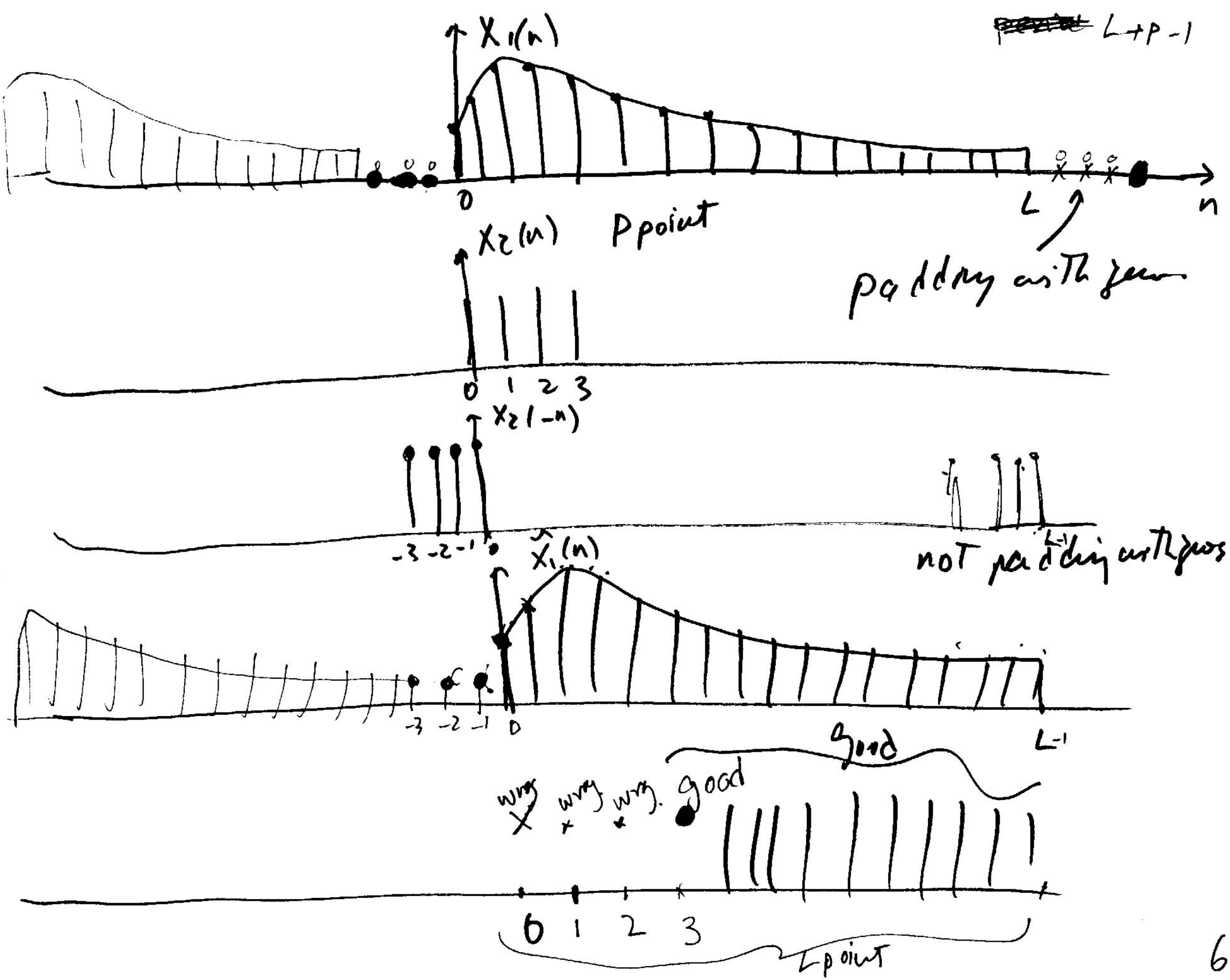
- Suppose I Take L pT DFT of x_1 & x_2.
  multiply Two, Lpt, DFTs.

take ~~[illegible]~~ L point IDFT.



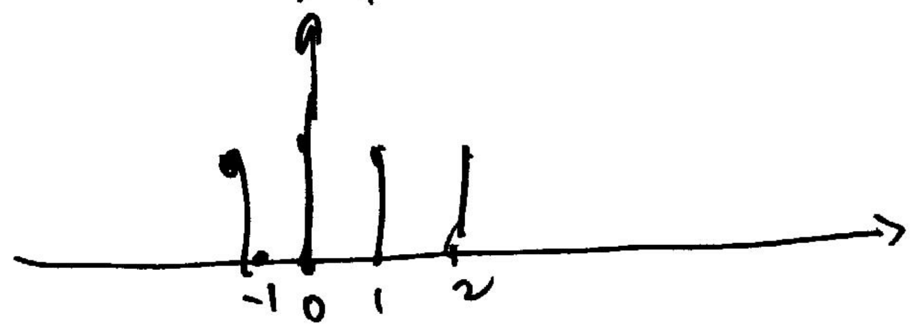
What goes wrong? Can show only the first $P-1$ points are wrong. Rest are correct.

- Why?. sketchy illustration

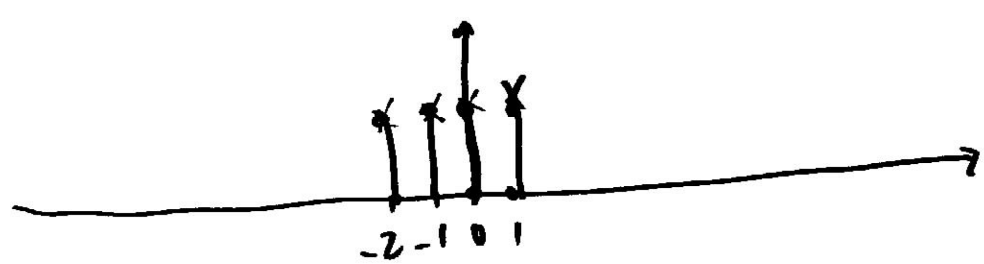

L+P-1
x1(n)
0
L
n
x2(n)
P point
padding with zeros
0 1 2 3
x2(-n)
-3 -2 -1 0
L-1
not padding with zeros
x̂1(n)
-3 -2 -1 0
L-1
good
wrong
wrong
wrong.
good
0 1 2 3
L point

$x_2(3-n)$

$x_2(2-n)$



$x_2(1-n)$



Overlap Save.

① Segment Sequence into L point chuncks. overlap with each other by P-1 points.

② L point circular Convolution of each chunk.

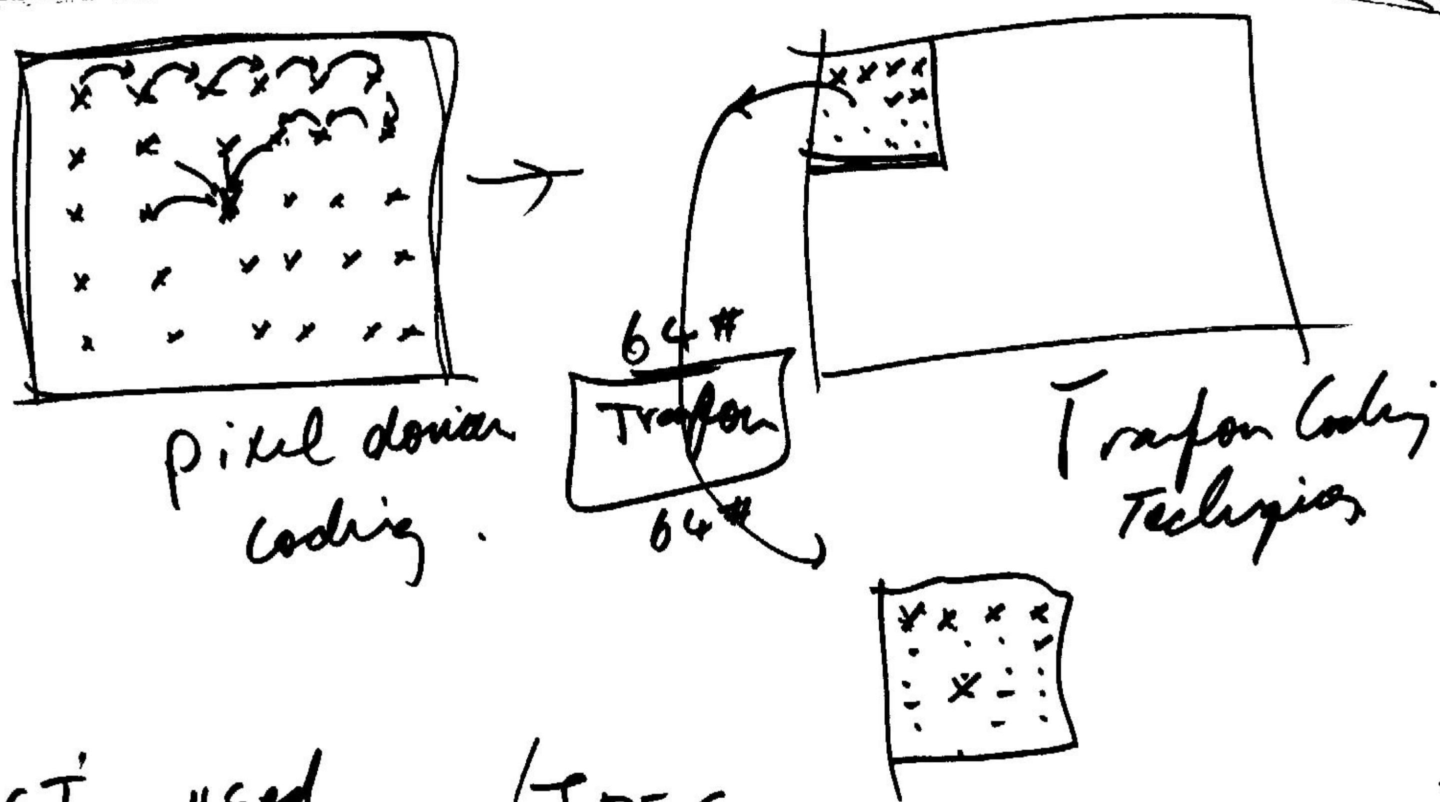
Multiply L pT DFT of chunk } → IDFT L pT.
L pT DFT of X2 }

③ Throw away first P-1 points of the answer in part 2. replace with previous segment.

Fig 8.23 of O&S
8.24 of O&S.

DCT = Discrete Cosine Transform.

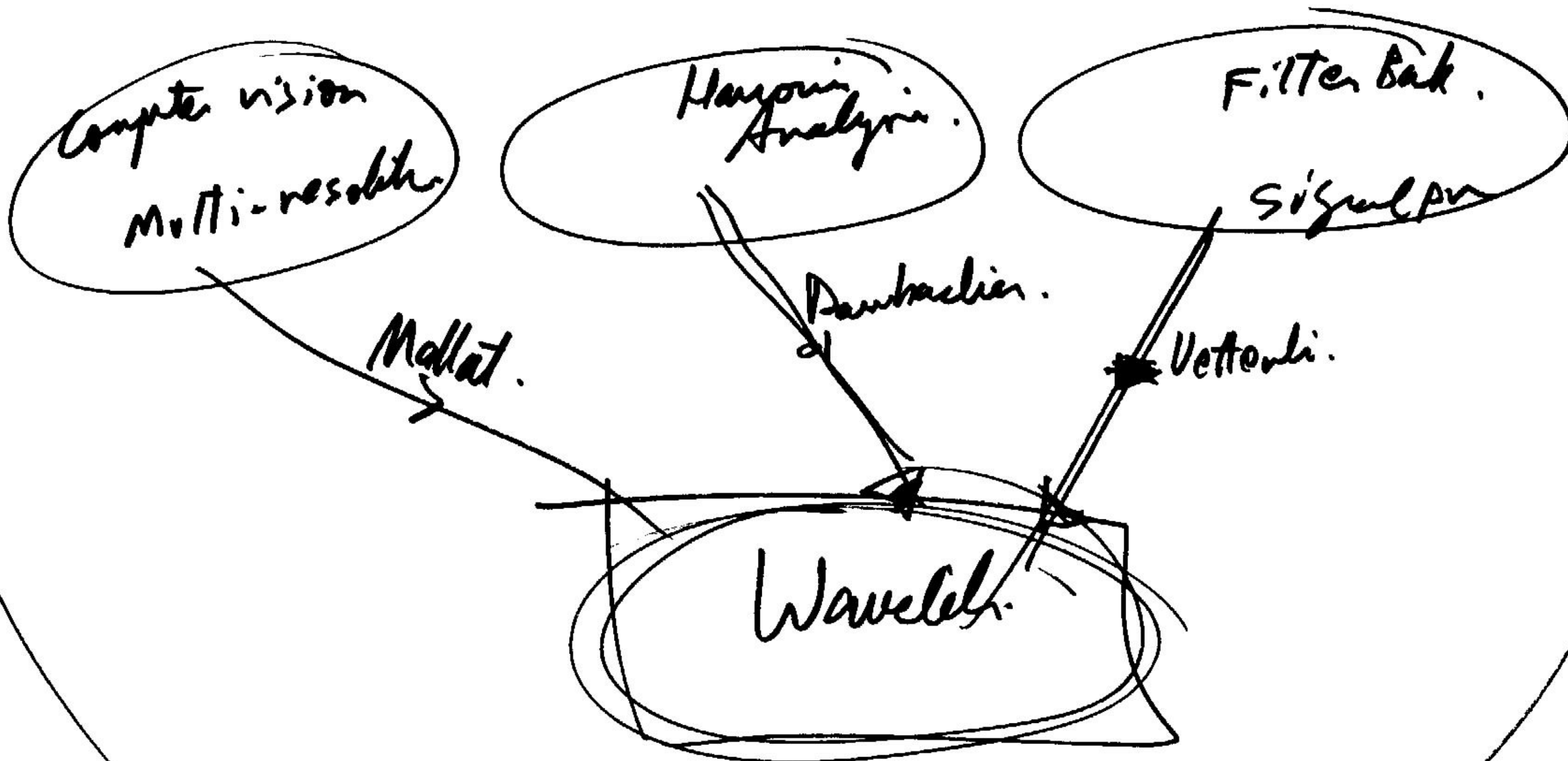
* Compaction property → coeffs stat. indep. of each other



* DCT used standard.

(JPEG,
MPEG 1
2
4
H.261, H.263, H.263+
H.263++, H.264, ...)

Other Than <u>JPEG 2000</u>, uses <u>wavelets</u>.
every other image / video stand uses DCT.



\+ Haar, Hartley, Haddamard, Walsh . — . . . .

$$x(n) \longrightarrow \boxed{\text{Real}} \longrightarrow \text{Real}.$$

$$\underset{\text{real}}{x(n)} \longrightarrow \boxed{\text{DFT}} \longrightarrow \underset{\text{complex.}}{X(k)}$$

Why DCT?

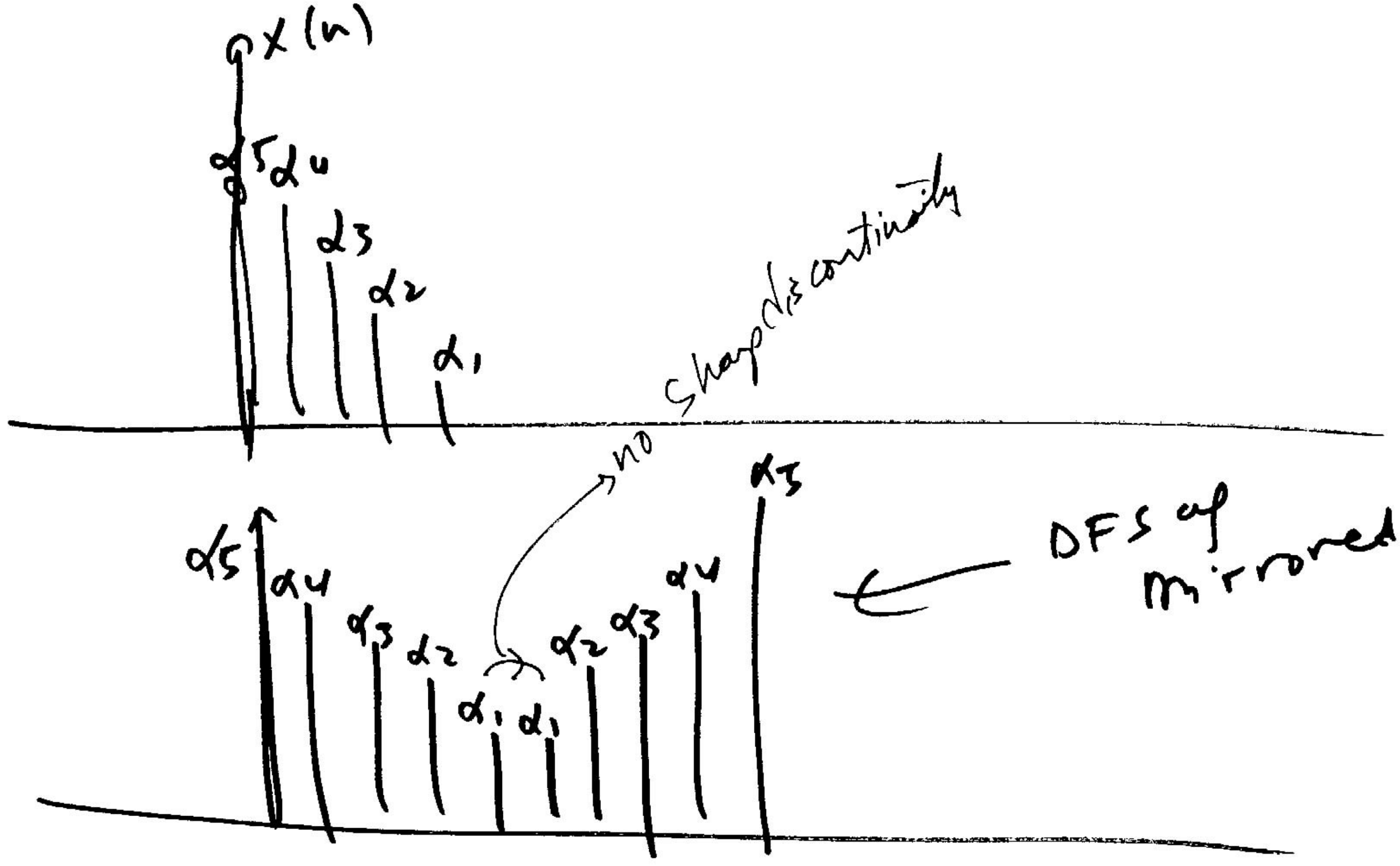
DFT: $x(n) \longrightarrow \tilde{x}(n) \xrightarrow{\text{DFS}} \tilde{X}(k) \xrightarrow{\text{one period}} \underset{\text{DFT}}{X(k)}$

expansion in Terms of $e^{j\frac{2\pi nk}{N}}$

sharp discontinuity

high freq.

Basic idea behind DCT is To replicate signal in a "better way". so that There is no sharp discontinuity.



4 Kinds of DCT

We'll focus DCT_2.

$$X^{C_2}(k) = 2 \sum_{n=0}^{N-1} x(n) \cos\left(\frac{\pi k(2n+1)}{2N}\right)$$

analysis

Synthesis:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} \beta(k) X^{C_2}(k) \cos\left(\frac{\pi k(2n+1)}{2N}\right)$$

$$\beta(k) = \begin{cases} \frac{1}{2} & k = 0 \\ 1 & 1 \le k < N \end{cases}$$

Relation DFT of $x(n)$ to its DCT-2.

proposal ①

1. $x(n)$. N pt seq. Real.
2. Take 2N pt DFT $\longrightarrow X(k)$
3. $2\,\mathrm{Re}\left\{ X(k)\, e^{-j\frac{2\pi k}{2N}} \right\}$

Proposal ②

1. start with N pt real seq $x(n)$
2. Pad it with N zeros $\rightarrow x_{2N}(n)$
3. form a periodic seq:

$$\tilde{x}_2(n) = \tilde{x}_{2N}(n) + \tilde{x}_{2N}(-n-1)$$

4. Take 2N pt DFT of one period of $\tilde{x}_2(n) \longrightarrow X_2(k)$