

HINT FOR

$$B_1 + B_2 + \dots + B_n.$$

$$\#33. E[X_1 X_2] = E\left[\left(\sum_i A_i\right) \left(\sum_j B_j\right)\right]$$

EE 126 DISC

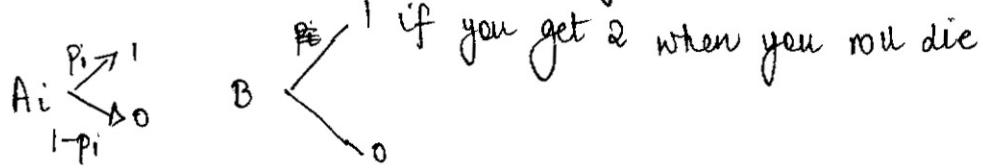
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①

$$A_1 + A_2 + \dots + A_n.$$

$$E[A_i B_i] = 0$$

$$E[A_i B_j] = E[A_i] \text{ Indep } E[B_j]$$



$$x \rightarrow$$

Pb 35 CHAPTER 4.

X, Y are 2 zero mean random variables i.e $E[X] = E[Y] = 0$

a) Show that X and $E[X|Y]$ are positively correlated

b) Show that the correlation co-efficients of Y and $E[X|Y]$ has the same sign as the correlation co-efficients of X and Y .

Solution:

$$\begin{aligned} a) \quad \text{cov}(X, E[X|Y]) &= E\left[(X - E[X]) \underbrace{(E[X|Y] - E[E[X|Y]])}_{0}\right] \\ &\quad \text{From law of iterated expr} \\ &= E[X E[X|Y]] \end{aligned}$$

To demonstrate it is a positive correlation, you might want to use law of iterated expressions

$$\begin{aligned} &= E\left[E\left[X \underbrace{E[X|Y]}_{\text{we conditioned}} | Y\right]\right] \\ &= E\left[E[X|Y] E[X|Y]\right] \\ &\quad (\text{we conditioned } E[X|Y] \text{ on } Y. \text{ Therefore it is a constant}) \end{aligned}$$

$$\therefore \text{cov}(X, E[X|Y]) = E\left[\underbrace{(E[X|Y])^2}_{>0}\right] > 0$$

$$\begin{aligned} b) \text{ cov}(Y, E[X|Y]) &= E[(Y - \underbrace{E[Y]}_0)(E[X|Y] - \underbrace{E[E[X|Y]]}_0)] \\ &= E[Y E[X|Y]] \end{aligned}$$

(given)

$E[X] = 0$
(law of iterated
expressions)

$$\begin{aligned} \text{Remember: } \text{cov}(X, Y) &= E[XY] \\ &= E[E[XY|Y]] \quad \leftarrow \begin{array}{l} \text{conditioning upon } Y \\ \text{to use law of} \\ \text{iterated expectation} \end{array} \\ &= E[Y E[X|Y]] \end{aligned}$$

$$\therefore \text{cov}(Y, E[X|Y]) = \text{cov}(X, Y)$$

Since the 2 covariances are equal, they have the same sign.

Bernoulli Process: a sequence of Bernoulli events.

Y_n is the # of trials up to and including n^{th} success.

$$P(Y_n = l) = P((n-1) \text{ 1's in } l-1 \text{ trials}) \cdot P(1 \text{ in } n^{\text{th}} \text{ trials})$$

$$= \binom{l-1}{n-1} p^{n-1} (1-p)^{l-1-(n-1)}$$

$$= \binom{l-1}{n-1} p^n (1-p)^{l-n}$$

Inter-arrival times : $T_1, T_2, T_3 \dots$ (time between 1's)

has a geometric distribution

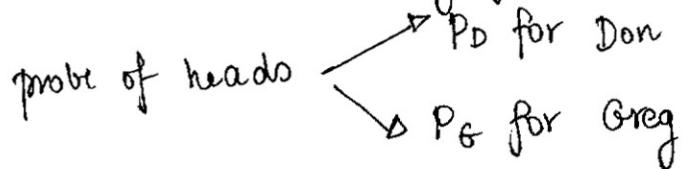
$$P(T_i = k) = (1-p)^{k-1} p.$$

$$Y_n = T_1 + T_2 + \dots + T_n$$

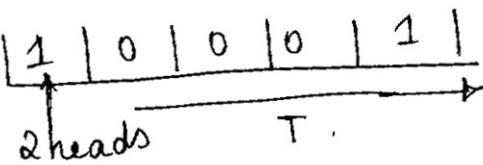
$$E[Y_n] = E[T_1] + E[T_2] + \dots + E[T_n] = \frac{n}{p}.$$

CHAPTER 5 #2

At each trial, Don and Greg flip biased coins



- a) Given that the flips on a particular trial resulted in 2 heads, find the PMF of the number of additional trials up to and including the next trial on which 2 heads result.

Soln: 

$$P(2 \text{ heads}) = P_D \cdot P_G$$

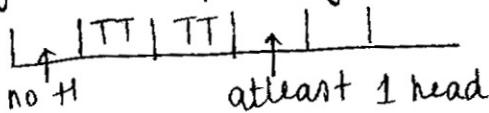
$$P(T=k) = (1 - P_D P_G)^{k-1} P_D \cdot P_G$$

- b) Given that the flips on a particular trial resulted in at least 1 head, find the probability that Don flipped a head on that trial

$$P(D_H | \text{at least 1 head}) = \frac{P(\#\text{H} \geq 1 | D_H) P(D_H)}{P(\#\text{H} \geq 1)}$$

$$= \frac{1 \cdot P_D}{P_D + P_G - P_D \cdot P_G} \quad (\Leftarrow P(A \cap B) = P(A) + P(B) - P(A \cup B))$$

- c) Starting from a trial on which no heads result, find the probability that Don's next flip of a head will occur before Greg's next flip of a head.



N is the trial on which you obtain H_1 for the first time.

(4)

$$P(A|N=k)$$

$\hat{\rightarrow}$ event Don flips head & Greg flips a tail given that there is 1 head.

Using Bayes Rule:

$$= \frac{P_D (1 - P_G)}{P_D + P_G - P_D \cdot P_G}$$

$$P(A) = \sum_{k=1}^{\infty} P(A|N=k) P(N=k)$$

$$= \frac{P_D (1 - P_G)}{P_D + P_G - P_D \cdot P_G} \sum_{k=1}^{\infty} P(N=k)$$