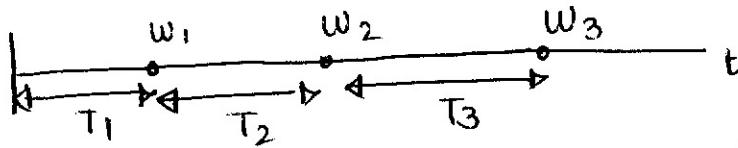


Renewal Process



w_i : arrival time

T_i : interarrival time

If Poisson process:

$$T_i \sim \exp(\lambda)$$

$$w_i \sim \text{erlang}(i, \lambda)$$

If renewal same picture

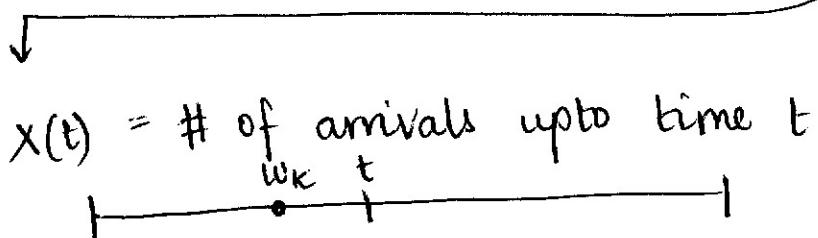
T_i IID

$$\text{eg } T_i \sim U[0, 1]$$

$$w_K = T_1 + T_2 + \dots + T_K$$

Let T_i have pdf f_T

$$\therefore f_{w_K}(t) = \underbrace{f_T * f_T * \dots * f_T}_{\text{convolution } K \text{ times}}$$



$X(t) = \# \text{ of arrivals upto time } t$

\downarrow

$$\text{iff } w_K \leq t \text{ iff } X(t) \geq K \Rightarrow P(X(t) \geq K) = P(w_K \leq t) = F_K(t)$$

if Poisson process $X(t) \sim P(2t)$

if renewal process \rightarrow don't know probability

$$P_{X(t)}(K) = P(X(t) \geq K+1)$$

$$= P(X(t) \geq K)$$

$$= F_{K+1}(t) - F_K(t)$$

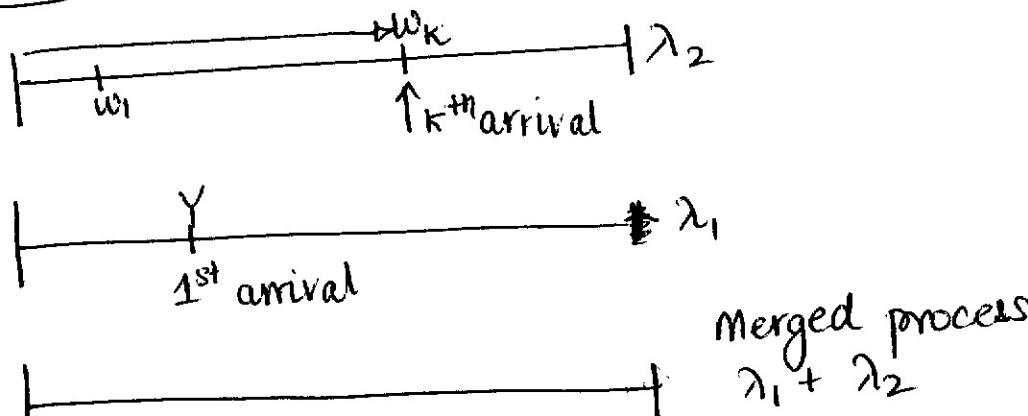
$$Y \sim \exp(\lambda_1)$$

$$W_K \sim \text{Erlang}(K, \lambda_2)$$

Y and W_K are independent

Let $M_K = \max(Y, W_K)$. Find a recursive formula for $E[M_K]$ in terms of $E[M_{K-1}]$

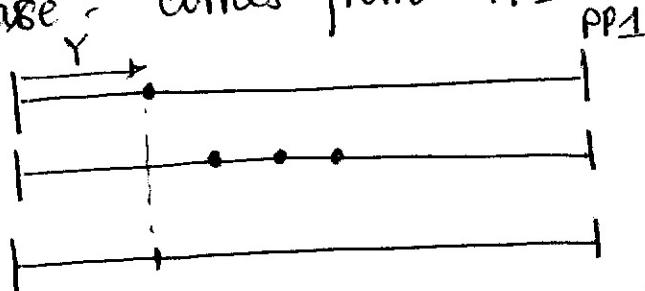
Solution:



For merged process?
the prob that the first arrival comes from PP1 is $\frac{\lambda_1}{\lambda_1 + \lambda_2}$
PP2 is $\frac{\lambda_2}{\lambda_1 + \lambda_2}$

condition on where the 1st arrival comes from is in the merged process

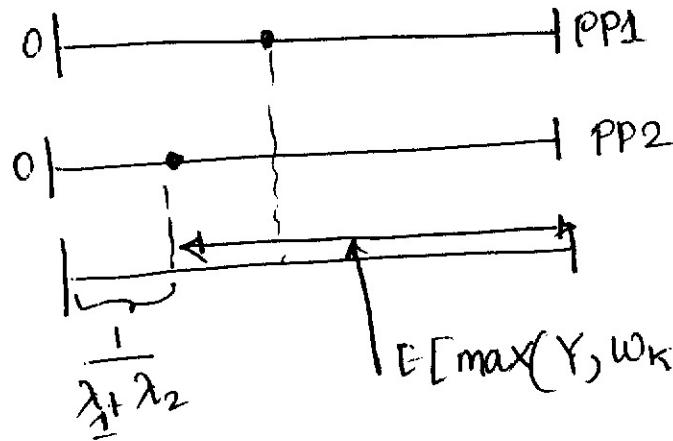
$E[M_K]$ $A =$ (Poisson Process 1)
First case: comes from PP1



$$E[\max(Y, W_K) | A] = \frac{1}{\lambda_1 + \lambda_2} + \frac{K}{\lambda_2}$$

↑ expectation of Erlang(K)

Θ : 1st arrival
comes from PP2



$$\therefore E[m_K] = \frac{\lambda_1}{\lambda_1 + \lambda_2} \left[\frac{1}{\lambda_1 + \lambda_2} + \frac{K}{\lambda_2} \right] + \frac{\lambda_2}{\lambda_1 + \lambda_2} \left[\frac{1}{\lambda_1 + \lambda_2} + E[m_{K-1}] \right]$$