Indp of Multiple Events

9/15/04

Event $A_1, A_2, \ldots, A_n$ are indp. if.

$$P\left(\bigcap_{i \in S} A_i\right) = \prod_{i \in S} P(A_i)$$

for <u>EVERY</u> subset S of $\{1, 2, 3, \ldots n\}$.

<u>EX</u> 3 events $A_1 \; A_2 \; A_3$

what are <u>ALL</u> possible subsets of $\{1, 2, 3\}$.

$\{1,2\}$ $\{1,3\}$ $\{2,3\}$, $\{1,2,3\}$

$$\left.\begin{aligned} P(A_1 A_2) &= P(A_1)\,P(A_2) \\ P(A_1 A_3) &= P(A_1)\,P(A_3) \\ P(A_2 A_3) &= P(A_2)\,P(A_3) \end{aligned}\right\} \rightarrow \text{pairwise independent.}$$

$$P(A_1 A_2 A_3) = P(A_1)\,P(A_2)\,P(A_3)$$

for us To call A_1 A_2 A_3 indp.

<u>EX</u> 2 indp. fair coin tosses.

HH
TH
HT
TT

$H_1 = \{$1st toss is heads$\}$

$H_2 = \{$2nd toss is heads$\}$

$D = \{$2 tosses have different results$\}$.

$\Rightarrow P(D) = 2/4 = \frac{1}{2}$

<u>Q</u>: Are H_1 H_2 and D indp?

H_1 & H_2 are indp. $P(H_1 H_2) = P(H_1)P(H_2)$.

Are H_1 and D indp?

$$P(D \mid H_1) = \frac{P(H_1 D)}{P(H_1)} = \frac{1/4}{\frac{1}{2}} = 1/2 = P(D)$$

$\Rightarrow$ D & H_1 are also indp.

How about $P(H_1 H_2 D) \overset{?}{=} P(H_1) \quad P(H_2) P(D)$

ϕ not True $\neq 0$

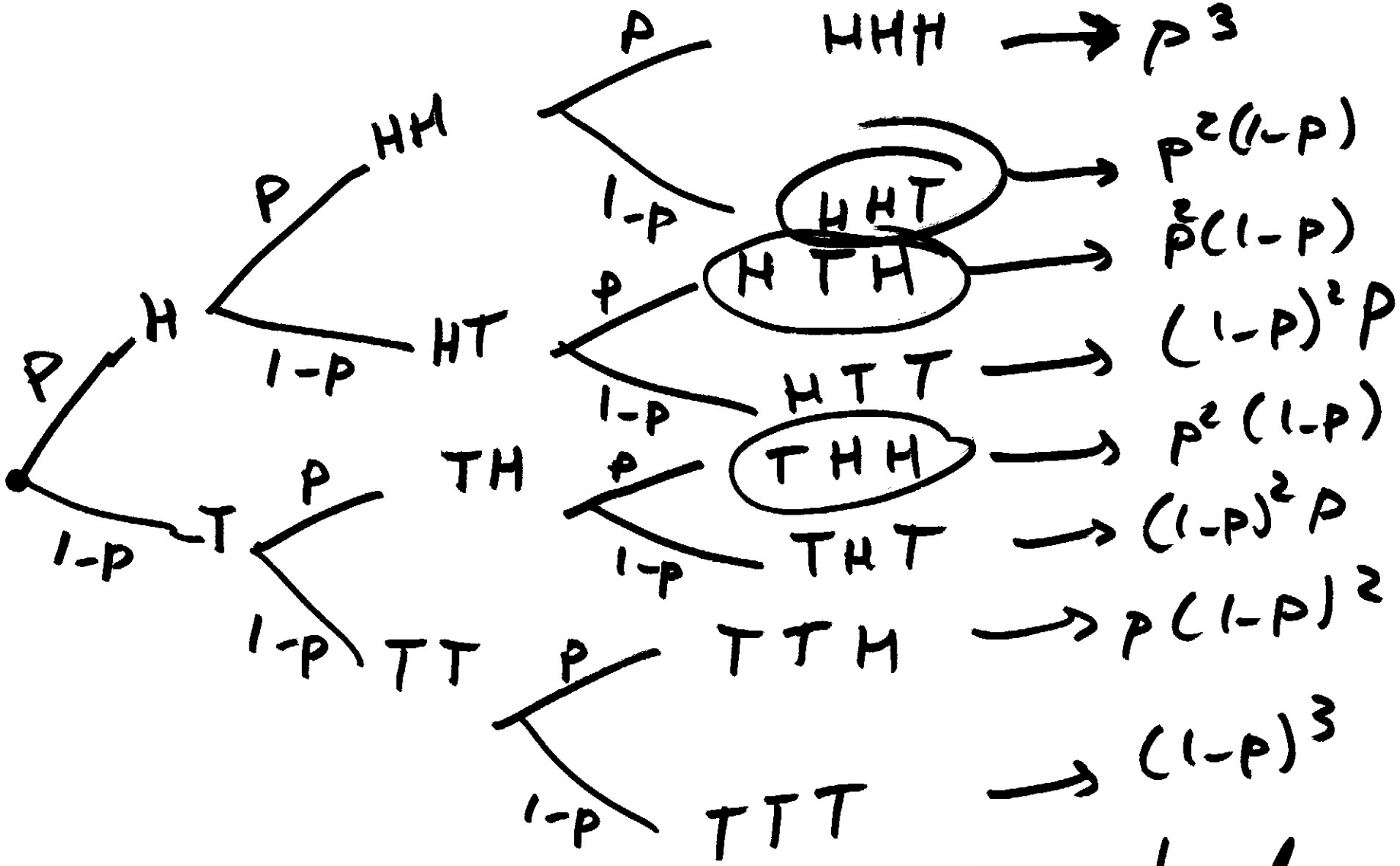
$\Rightarrow$ $H_1, H_2,$ and D are not indep.

Independent Trials

- Seq of independent, but identical stages in an experiment.

- If 2 outcomes $\longrightarrow$ Bernoulli Trials.

Ex n indp Tosses of a coin.



Q What is prob that I get K heads in a seq. of n trials.

for one seq of k heads and n-k Tails

$$P(\cdot) = p^k (1-p)^{n-k}$$

How many such seq are There?

$$\binom{n}{k} \triangleq \frac{n!}{k!\,(n-k)!} = \binom{n}{n-k}$$

n choose k

n choose n-k

Ex $n=3$ $k=2$ $\frac{3!}{2!\,1!} = 3$

$$P(k \text{ heads}) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \frac{n!}{k!\,(n-k)!} p^k (1-p)^{(n-k)}$$

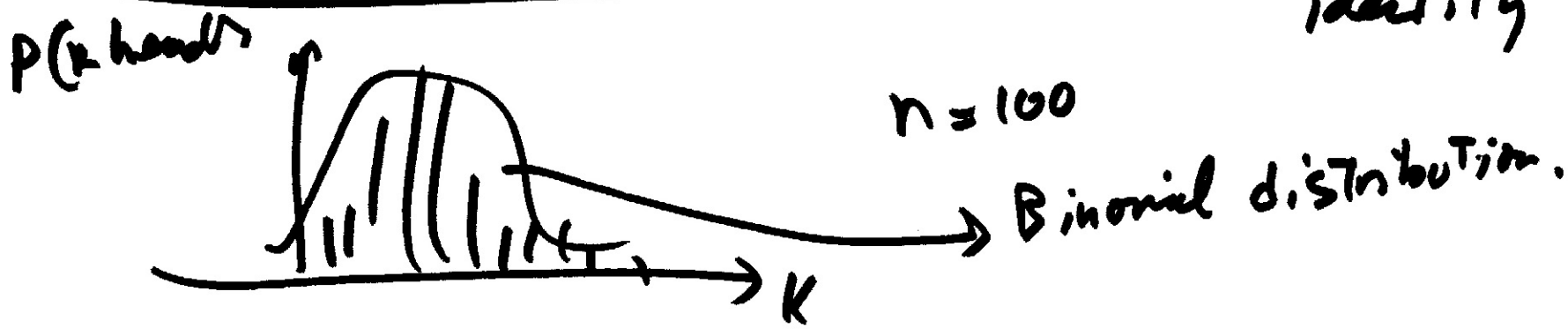
binomial prob.

binomial coeff.

$$\sum_{k=0}^{n} P(k \text{ heads}) = 1 \implies$$

$$\boxed{\sum_{k=0}^{n} \frac{n!}{k!\,(n-k)!} p^{k} (1-p)^{n-k} = 1}$$

identity



EX

$$\frac{100{,}000{,}000}{4{,}000} = 25{,}000$$

$$\frac{100{,}000}{4{,}000} = 25$$

0.001

~~0.01~~

~~0.001~~

~~0.0001~~

Q: what is prob

Ex

c modems $\qquad c << n$

n subscribers

each customer dials up with prob (p) indep. of the other.

Q: what is prob that customers get rejected?

Define $p(k) = p(\ k$ subs are connected$)$

$$p(\cdot) = \sum_{k=C+1}^{n} p(k) \quad \leftarrow = p(C+1) + p(C+2) \ldots\ldots =$$

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$p(\cdot) = \sum_{k=C+1}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$

<u>Ex</u> $n = 100$ $\quad p = 10\%$ $\quad C = 15$

$\Rightarrow p(\cdot) = 0.0399$

Counting

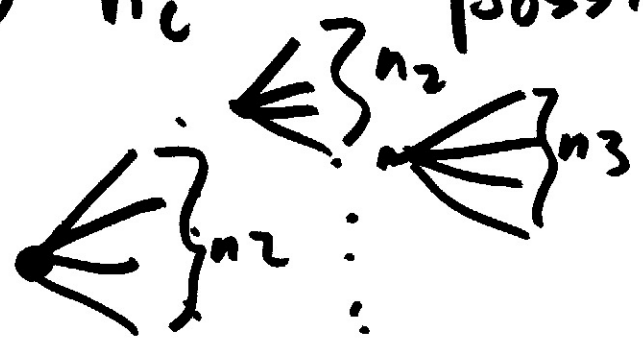
- helps us do prob. calc.

Process r stages.

① n_1 possible results in stage 1

② For **ALL** possible results of 1st stage, There are n_2 possible results at stage 2

③ n_i possible results at stage i



Total # of possibilities $= n_1 n_2 \cdots n_r$

Ex 7 digit phone #:

$$\underset{1}{\underline{10}} \times \underset{2}{\underline{10}} \times \underset{3}{\underline{10}} \times \underset{4}{\underline{10}} \times \underset{5}{\underline{10}} \times \underset{6}{\underline{10}} \times \underset{7}{\underline{10}} = 10^7$$

if no 0 or 2 in 1st digit

$$\underset{\downarrow}{8} \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 8 \times 10^6$$

Selecting k objects out of n

Order matters → permutation

order doesn't matter → combination

Ex 4 balls

#1, #2, #3, #4.

Choose 2 out of 4 balls, how many diff. sequences can I get?

in draw #1 & #2.

order matter.

1 2
1 3
1 4
2 3
2 4
2 1
3 4
3 1
3 2
4 1
4 2
4 3

12

$4 \times 3 = 12$

n objects choose k

① first Time 1 out of n
left with n-1 objects.

② 2nd Time 1 out of n-1

③ 3rd 1 out of n-2

n

$\vdots$

$n-1$

$n-2$

$\vdots$

$n-k+1$

kth

$= n \times (n-1) \times (n-2) \ldots (n-k)$ # of permutations.

$$= \frac{n(n-1)(n-2)\ldots(n-k+1)(n-k)(n-k-1)\ldots\ldots 2 \cdot 1}{(n-k)(n-k-1)(n-k-2)\ldots\ldots 2 \cdot 1}$$

$$= \frac{n!}{(n-k)!}$$

n objects, choose k.

$$\frac{n!}{(n-k)!}$$

<u>Ex</u> How many distict 4 letter words are there.

$$\frac{26!}{22!} = 26 \times 25 \times 24 \times 23$$

$$\approx 358000$$