UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probablity and Random Processes

Problem Set 4
Fall 2006

Issued: Thursday, September 21, 2006 **Due:** Friday, September 29, 2006

Reading: For this problem set: Chapter 2

Problem 4.1
A group of entrepreneurs just purchased troubled Air Confusion. Air Confusion currently only offers service to Reykjavik and Auckland. Because they are so disorganized, flights occur in a random manner and their planes often crash. The probability that a flight to Reykjavik or Auckland crashes is 1/5 and 1/10, respectively. Any particular flight goes to Reykjavik with probability 2/3 and Auckland with probability 1/3.

(a) What is the probability that a randomly chosen flight crashes?

(b) What is the expected number of flights before the first crash?

(c) What is the expected number of flights that occur before the first crash and after 3 non-crash flights?

(d) Air Confusion has 1000 flights per year. What is the probability that they have 1 or less crash in a year?

(e) Suppose that the entrepreneurs discover that if 100 new mechanics are hired, the probability of a safe flight on any particular Air Confusion flight will be 0.9999. Using the Poisson approximation (see p. 79 in textbook), what is the probability that all 1000 flights in a given year arrive safely at their destination?

Problem 4.2
Your probability class has 250 undergraduate students and 50 graduate students. The probability of an undergraduate (or graduate) student getting an A is 1/3 (or 1/2, respectively). Let X be the number of students that get an A in your class.

(a) Find the PMF of X.

(b) Calculate $\mathbb{E}[X]$ using the total expectation theorem, rather than the PMF of X.

(c) Calculate $\mathbb{E}[X]$ and $\mathrm{var}(X)$ by viewing X as a sum of random variables, whose statistics are easily calculated.

Problem 4.3
The joint PMF of discrete random variables X and Y is given by

$$p_{X,Y}(x,y) = \begin{cases} Cx^2\sqrt{y}, & \text{if } x = -5, -4, \ldots, 4, 5 \text{ and } y = 0, 1, \ldots, 10 \\ 0 & \text{otherwise.} \end{cases}$$

Here, C is some constant. What is $\mathbb{E}[XY^3]$? (*Hint*: This question admits a short answer/explanation. Don't spend too much time doing calculations.)

Problem 4.4

In this problem, the result established in part (a) will be helpful to answer parts (b) and (c).

(a) Suppose a random variable X can have only non-negative integer values. Show that $\mathbb{E}[X] = \sum_{x=0}^{\infty} \mathbb{P}(X > x)$.

(b) An urn contains b blue and r red balls. Balls are removed at random until the first blue ball is drawn. Show that the expected number of balls that are drawn is $(b+r+1)/(b+1)$.

Hint: You may find the following combinatorial identity useful, which can be proved (though you need not do so) by using the binomial theorem or repeated application of the simple identity $\binom{x}{r-1} + \binom{x}{r} = \binom{x+1}{r}$:

$$\sum_{x=0}^{r} \binom{x+b}{b} = \binom{r+b+1}{b+1} \quad .$$

(c) Assume the balls removed in part (b) are replaced so that the urn again contains b blue balls and r red balls. Now, instead of removing balls until the first blue ball is drawn, consider the experiment where balls are removed at random until all the balls remaining in the urn are of the same color. Find the expected number of balls remaining in the urn.

Problem 4.5

Joe Lucky plays the lottery on any given week with probability p, independently of whether he played on any other week. Each time he plays, he has a probability q of winning, again independently of everything else. During a fixed time period of n weeks, let X be the number of weeks that he played the lottery and Y the number of weeks that he won.

(a) What is the probability that he played the lottery any particular week, given that he did not win anything that week?

(b) Find the conditional PMF $p_{Y|X}(y \mid x)$.

(c) Find the joint PMF $p_{X,Y}(x, y)$.

(d) Find the marginal PMF $p_Y(y)$. *Hint:* One possibility is to start with the answer in (c), but the algebra can be messy. But if you think intuitively about the procedure that generates Y, you may be able to guess the answer.

(e) Find the conditional PMF $p_{X|Y}(x \mid y)$.

In all parts of this problem, make sure to indicate the range of values to which your PMF formula applies.