UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: PROBABLITY AND RANDOM PROCESSES

Problem Set 3

Fall 2007

Issued: Thursday, September 13, 2007 **Due:** Friday, September 21, 2007

Reading: Bertsekas & Tsitsiklis, §1.4, §1.5. and §1.6

Problem 3.1
A mouse is wandering on a rectangular grid with grid points $(a, b), a, b = 0, 1, 2, 3, \ldots$. From grid position (a, b), the mouse must move either to position $(a + 1, b)$ or $(a, b + 1)$.

(a) Find the total number of different paths from $(0, 0)$ to (n, n).

(b) Find the total number of different paths from $(1, 0)$ to $(n + 1, n)$ that pass through at least one of the points (r, r), with $1 \leq r \leq n$.

Hint: Consider counting instead the number of unrestricted paths from $(0, 1)$ to $(n + 1, n)$. By thinking about reflection across the diagonal, show that the number of such unrestricted paths is the same as the number of paths to count in part (b).

Problem 3.2
A lazy GSI returns exams to students randomly. If the class has n students, what is the probability that at least one student receives his/her own exam? Compute the limiting value of this probability as $n \to +\infty$.

Hint: The inclusion-exclusion formula (p. 54, textbook) could be useful. You can also write a program to verify your expression via experimentation.

Problem 3.3
(Fishing trip) Koichi's backyard pond contains g goldfish and c catfish. All fish are equally likely to be caught.

(a) Suppose that Koichi catches a total of k fish (no fish are thrown back). What is the probability of catching x goldfish?

(b) Now suppose that all k fish are returned to the pond, and he starts fishing again, and catches a total of m fish. What is the probability that among the caught set of m fish, exactly 2 goldfish are included that were also caught in the first catch? (Assume that fish do not learn from experience.)

Problem 3.4
In each running of a lottery, a sequence r numbers are drawn independently (with replacement) from a total of n numbers. You win the lottery if the r-number sequence on your ticket matches the drawn sequence.

(a) Suppose I buy one (randomly chosen) lottery ticket. What is the probability of winning the lottery?

(b) Suppose that I hack into the lottery computer, and program it to only draw sequences with r numbers in a non-decreasing order. I then go and buy a ticket with its numbers in non-decreasing order. Now what is my probability of winning the lottery?

Hint: Let $(x_1, x_2, \ldots, x_m)$ be a collection of integers with each $x_i \geq 0$ such that $\sum_{i=1}^{m} x_i = k$. The number of such collections is $\binom{m+k-1}{m-1}$.

Problem 3.5

An urn contains a aquamarine balls, and b blue balls. The balls are removed successively without replacement until the urn is empty. If $b > a$, show that the probability that at all stages until the urn is empty there are more blue than aquamarine balls in the urn is equal to $(b-a)/(b+a)$.

Problem 3.6

A communication system transmits one of three signals, s_1, s_2 and s_3, with equal probabilities. The transmission is corrupted by noise, causing the received signal to be changed according to the following table of conditional probabilities:

		Receive, j		
	$\mathbb{P}(s_j \vert s_i)$	s_1	s_2	s_3
	s_1	0.3	0.4	0.3
Send, i	s_2	0.002	0.99	0.008
	s_3	0.8	0.15	0.05

For example, if s_1 is sent, the probability of receiving s_3 is 0.3. The entries of the table list the probability of s_j received, given that s_i is sent, i.e., $P(s_j \text{ received}|s_i \text{ sent})$.

(a) Compute the (unconditional) probability that s_j is received for $j = 1, 2, 3$.

(b) Compute the probability $\mathbb{P}(s_i \text{ sent}|s_j\text{received})$ for $i, j = 1, 2, 3$.

(c) As seen from the numbers above, the transmitted signal can be very different from the received signal. Thus, when symbol s_j is received, it may not be a good idea to conclude that s_j was sent. We need a decision scheme to decide which signal was sent based on the signal we receive with the lowest possible *error probability*, that is, the probability of making a wrong decision. Find the scheme that minimizes the overall error probability.

Problem 3.7

Evelyn has n coins in her left pocket, and n coins in her right pocket. Every minute, she takes a coin out of a pocket chosen at randomly (and independently) from previous draws, until the first time one of her pockets is empty. For an integer $t \in [1, n]$, compute the probability of having t coins in the other pocket when she stops.