UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probablity and Random Processes

Problem Set 6
Fall 2007

Issued: Thursday, October 11, 2007 **Due:** Friday, October 19, 2007

Reading: Bertsekas & Tsitsiklis, §3.4–3.6

Problem 6.1
Let X have a uniform distribution in the unit interval $[0, 1]$, and let Y have an exponential distribution with parameter $\lambda = 2$. Assume that X and Y are independent. Let $Z = X + Y$.

(a) Find $P(Y \geq X)$.

(b) Find the conditional PDF of Z given that $Y = y$.

(c) Find the conditional PDF of Y given that $Z = 3$.

Problem 6.2
Random variable X is uniformly distributed in the region $1 \leq x \leq 2$. Determine the PDF $f_Y(y)$ for all values of y if:

(a) $Y = e^{-2X}$

(b) $Y = (X - 1.2)^2$

Problem 6.3
Consider the following problem and a purported solution. Either declare the solution to be correct or explain the flaw.
Question: Let X and Y have the joint density

$$f_{X,Y}(x, y) = \begin{cases} 1, & x \in [0, 1] \quad \text{and} \quad y \in [x, x + 1]; \\ 0, & \text{otherwise.} \end{cases}$$

Find $f_X(x)$, $f_Y(y)$, and $f_{Y|X}(y|x)$. Are X and Y independent?
Solution:

$$f_X(x) = \int f_{X,Y}(x, y)\, dy = \int_x^{x+1} 1 \cdot dy = 1.$$

$$f_Y(y) = \int f_{X,Y}(x, y)\, dx = \int_0^1 1 \cdot dx = 1.$$

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x, y)}{f_X(x)} = \frac{1}{1} = 1.$$

Since $f_{Y|X}(y|x)$ does not depend on y, we have that X and Y are independent. Alternatively, X and Y are independent because $f_{X,Y}(x, y) = f_X(x) f_Y(y)$.

Problem 6.4
Alice and Bob work independently on a problem set. The time for Alice to complete the set is exponentially distributed with mean 4 hours. The time for Bob to complete the set is exponentially distributed with mean 6 hours.

(a) What is the probability that Alice finishes the problem set before Bob?

(b) Given that Alice requires more than 4 hours, what is the probability that she finishes the problem set before Bob?

(c) What is the probability that one of them finishes the problem set an hour or more before the other one?

Problem 6.5

A river has a bridge every 8 miles. You would like to cross the river by bridge, but you are currently on a road that runs parallel to the river and is separated from the river by a dense forest. Occasionally there is a straight path through the forest which runs perpendicular to the river and the road. Assume you start walking on the road at a point such that after walking 2 miles, you are directly opposite a bridge (so if you found a path after walking 2 miles, you would emerge from the forest at a bridge). Let R be the distance you walk on the road before you find a path through the forest. Let S be the distance you walk on the river bank to reach a bridge, assuming you head for the nearest bridge when you emerge from the forest. Find the PDF of S for the following cases.

(a) the PDF $f_R(r)$ is nonzero in the interval $[0, 12]$ and zero everywhere else.

(b) the random variable R is exponentially distributed with mean 1 mile.

Problem 6.6

An absent-minded professor schedules two student appointments for the same time; unfortunately, the professor is only able to meet with one student at a time. The appointment durations are independent and exponentially distributed with mean thirty minutes. Determine the expected value and the variance of the time between the arrival of the first student and the departure of the second student for each of the two following cases:

(a) The first student arrives on time, but the second student arrives 5 minutes late. (The expected value is NOT 60 minutes nor 65 minutes.)

(b) The first student arrives on time, but the second student arrives X minutes late, where X is exponential with mean 5 min.