UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS126: Probability in EECS

Problem Set 8
Fall 2013

Issued: Tuesday, October 22, 2013 **Due:** In class Thursday, October 29, 2013

Problem 1. Consider the random walk example in the notes, page 74. Derive the Kalman filter equations and implement it in Matlab to reproduce Figure 4.15.

Problem 2. Let $(X, Y)^T \sim N([0; 0], [3, 1; 1, 1])$. Find $E[X^2|Y]$.

Problem 3. Let $(X, Y, Z)^T \sim N([0; 0; 0], [5, 3, 1; 3, 9, 3; 1, 3, 1])$. Find $E[X|Y, Z]$.

Problem 4. Consider arbitrary random variables X and Y. Prove the following property.

$$\mathrm{Var}(Y) = \mathrm{E}(\mathrm{Var}(Y \mid X)) + \mathrm{Var}(\mathrm{E}(Y \mid X)).$$

Problem 5. Let the joint density of two random variables X and Y be

$$f_{X,Y}(x, y) = \frac{1}{4}(2x + y)1\{0 \le x \le 1\}1\{0 \le y \le 2\}.$$

First show that this is a valid joint distribution. Suppose you observe Y drawn from this joint density. Find $\mathrm{MMSE}[X|Y]$.

Problem 6. Given four independent $N(0, 1)$ random variables X, Y, Z, and V, find the following minimum mean square estimator:

$$E[X + 2Y + 3Z|Y + 5Z + 4V]$$

Find the mean squared error of the estimator.