UC Berkeley
Department of Electrical Engineering and Computer Sciences
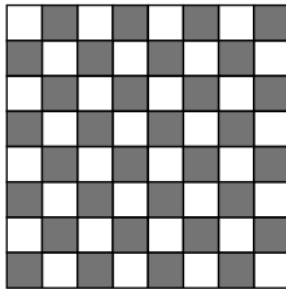
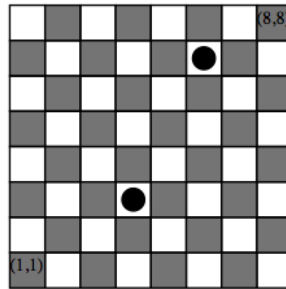EE126: Probability and Random Processes

**Discussion 1**
Fall 2014

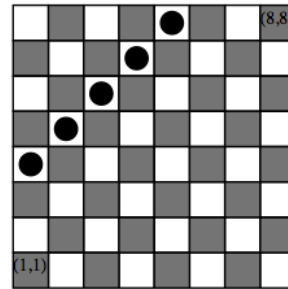**Date:** Wednesday, September 3, 2014

***Problem*** 1. Consider the following chess boards. Note that a rook can move either one step to the right or one step upwards.



(i)　　　　　　　　(ii)　　　　　　　　(iii)

(a) How many ways are there to place 8 pawn pieces on a chess board?

(b) How many ways are there to place 8 pawn pieces so that no pair shares the same row or column?

(c) How many ways are there to place 6 pawn pieces and 2 rook pieces so that the rooks are in adjacent columns?

(d) Suppose that two positions on the chessboard are occupied (see Figure (ii)). How many different paths can the rook take from (1,1) to (8,8) without going through either of the two occupied positions?

(e) Now suppose that five positions are blocked, as shown in Figure (iii). How many different paths from (1, 1) to (8, 8) does the rook now have?

**Problem** 2. There are $n$ urns of which the $r$th contains $r - 1$ red balls and $n - r$ blue balls. You pick an urn at random and remove two balls at random.

(a) Find the probability that the second ball is blue.

(b) Find the probability that the second ball is blue, given that the first is blue.

**Problem** 3. $N$ couples enter a casino. After two hours, $N$ of the original $2N$ people remain (the rest have left). Each person decides to leave with probability $p$ independent of others' decisions. What is the expected number of couples still in the casino at the end of two hours?

(a) Try to solve this problem without following the steps described below.

(b) Describe the event space conditioned on the fact that exactly $N$ people remained.

(c) What is the probability of a particular couple remaining?

(d) (Indicator) Now, consider a new random variable that takes value of 1 if the particular couple remains, and takes 0 otherwise. What is the expected value of this random variable?

(e) Express the number of couple in another form using indicators.

(f) Find the expected number of couples still in the casino at the end of two hours