UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion 9
Fall 2014

Date: Wednesday, October 29, 2014

Problem 1. (Matrix completion and Random Bipartite Graphs) Matrix completion is a problem of filling missing entries given the incomplete matrix has a low rank. For this problem, consider a matrix $A \in \mathcal{R}^{n \times n}$ of rank 1.

(a) Consider the following matrix A of rank 1. Can you 'uniquely' complete the following matrix? If so, fill the matrix.

$$A = \begin{bmatrix} 3 & ? & ? \\ ? & 5 & 9 \\ 6 & ? & 18 \end{bmatrix}$$

(b) Consider the following matrix B of rank 1. Can you 'uniquely' complete the following matrix? If so, fill the matrix.

$$B = \begin{bmatrix} 3 & ? & 9 \\ ? & 5 & ? \\ 6 & ? & 18 \end{bmatrix}$$

(c) Consider a bipartite graph with n 'row nodes' and n 'column nodes'. Draw an edge between row node i and column node j if $A(i, j)$ is known. Can you find the difference between two bipartite graphs corresponding to the above matrices?

(d) Prove that if an underlying bipartite graph is not connected, one cannot uniquely complete the matrix.

(e) (Random matrix model, converse) [Optional] Assume that each element of a matrix is revealed with probability p independenty of others. What is the minimum p required to complete random matrices?

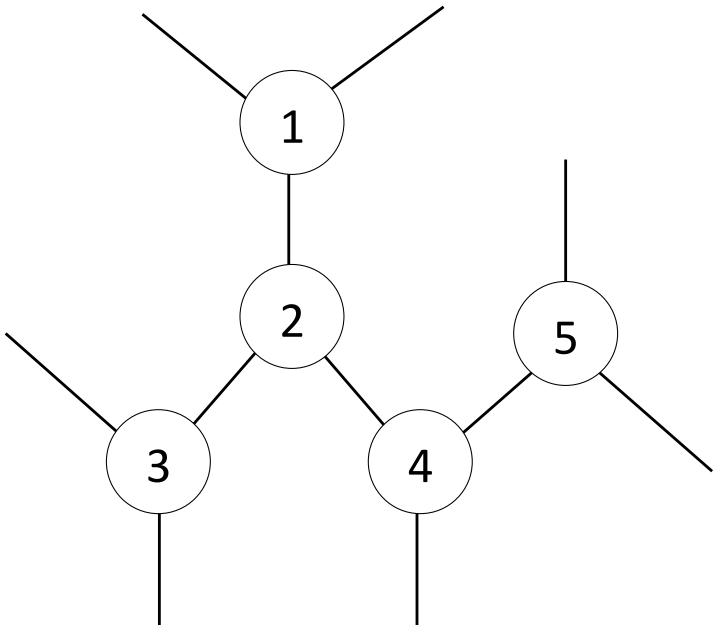
(f) (Random matrix model, achievability) [Optional] Can you find an algorithm that can complete all matrices whose the underlying graph is connected? How would you prove it?

Problem 2. (Infection source detection) Consider a graph where each node represent each person and edges represent connectivity between them. At time 1, the source of the rumor u^* appears. At time 2, the source chooses one of its neighbors, and infects the chosen neighbor. Similarly, in the following time slots, one of the uninfected nodes that are neighboring the nodes that are already infected in the previous time slots is chosen uniformly at random, and get infected. Right after time n, you observe the infected network with n infected nodes, and you want to detect the source of the infection.

(a) Consider an infinitely long linear-network: node i is connected with node $(i-1)$ and node $(i+1)$ for all $i \in \mathbb{Z}$.

At time 11, 11 nodes, $\{-5, -4, \ldots, 4, 5\}$, are infected. Find the MLE of the source of the infection.

(b) Consider the following infection graph: at time 5, the following 5 nodes are infected. Find the MLE of the source of the infection.



(c) Consider the same graph. Given that node 4 has twice higher probability of being the source than the others, find the MAP estimate of the source of the infection.

(c) Consider an infinitely large 2D grid: node (i, j) is connected with node $(i+u, j+v)$ for all $(u, v) \in \{(\pm 1, 0), (0, \pm 1)\}$.

At time 4, 4 nodes $\{(0,0), (1,0), (0,1), (-1,0)\}$ are infected. Find the MLE of the source of the infection. Which node is the second most likely source?