

Problem Set 1

Fall 2014

Issued: Thursday, August 28, 2014

Due: Thursday, September 04, 2014

Problem 1. Let A and B be events with probabilities $P(A) = \frac{4}{5}$ and $P(B) = \frac{1}{3}$. Show that $\frac{2}{15} \leq P(A \cap B) \leq \frac{1}{3}$, and give examples to show that both upper and lower bound are tight. Find corresponding bounds for $P(A \cup B)$.

Problem 2. Find an example of 3 events A , B , and C such that each pair of them are independent, but they are not mutually independent. Show the calculations.

Problem 3. Alice and Bob want to choose between the opera and the movies by tossing a fair coin. Unfortunately, the only available coin is biased (though the bias is not known exactly). How can they use the biased coin to make a decision so that either option is equally likely?

Problem 4. We deal from a well-shuffled 52-card deck. Calculate the probability that the 13th card is the first king.

Problem 5. Alice has a high fever and go to the doctor to identify the cause. 2% of the people have H1N1, 15% of the people have the flu, and the rest have neither. Suppose that 100% of the H1N1 people have a high fever, 35% of the flu people have a high fever, and 1% of the people who have neither, have a high fever. Is it more likely that Alice has H1N1, the flu, or neither?

Problem 6. Figure 1 is the reliability graph of a system. The links of the graph represents components of the system. Each link i is working with probability p_i and defective with probability $1 - p_i$, independently of the other links. The system is operational if the nodes S and T are connected. Thus, the system is built of two redundant subsystems. Each subsystem consists of a number of components. Calculate the probability that the system is operational.

Mini-Lab. Download [Lab1 - iPython Tutorial.ipynb](#) from course websites. Complete the iPython tutorial by filling missing code blocks. Submit your `ipynb` file online. (Note: Since this the first mini-lab, and it's tutorial in nature, you don't need to self-grade this week's mini-lab.)

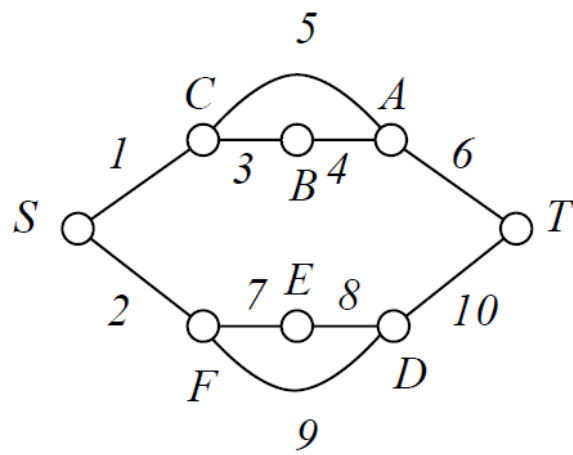


Figure 1: reliability graph