UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 11
Fall 2014

Issued: Thursday, November 20, 2014 **Due:** Thursday, December 4, 2014

Problem 1. Let $(V_n,\ n \geq 0)$ be i.i.d. $N(0, \sigma^2)$ and independent of $X_0 = N(0, u^2)$. Define

$$X_{n+1} = aX_n + V_n,\ n \geq 0.$$

(a) What is the distribution of X_n for $n \geq 1$?

(b) Find $E(X_{n+m}|X_n)$ for $0 \leq n < n + m$.

(c) Find u so that the distribution of X_n is the same for all $n \geq 0$.

Problem 2. Let θ be uniform random variable in $[0, 1]$, and given θ, random variable X is uniformly distributed in $[0, \theta]$. Find $E[\theta|X]$.

Problem 3. Let $(X, Y, Z)^T \sim N(\mu, \Sigma)$, and

$$\mu = [0, 0, 0]^T,$$

and

$$\Sigma = \begin{pmatrix} 5 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}.$$

Find $E[X|Y, Z]$.

Problem 4. Let the joint density of two random variables X and Y be

$$f_{X,Y}(x, y) = \frac{1}{4}(2x + y)1\{0 \leq x \leq 1\}1\{0 \leq y \leq 2\}.$$

First show that this is a valid joint distribution. Suppose you observe Y drawn from this joint density. Find MMSE$[X|Y]$.

Problem 5. Given four independent $N(0, 1)$ random variables X, Y, Z, and V, find the following minimum mean square estimator:

$$E[X + 2Y + 3Z|Y + 5Z + 4V]$$

Find the mean squared error of the estimator.

Problem 6. Let X, Y, Z be three random variables. Prove formally that

$$E(|X - E[X|Y]|^2) \geq E(|X - E[X|Y, Z]|^2).$$

What is the intuition behind the inequality?

Mini-Lab 1. Download `Lab11 - Viterbi.ipynb` from course websites. Complete the mini-lab by filling missing code blocks, and working on problems. Submit your `ipynb` file and `pdf` file online.

Mini-Lab 2. (Optional) Download `Lab12 - RNA Sequencing.ipynb` from course websites. Complete the mini-lab by filling missing code blocks, and working on problems. Submit your `ipynb` file and `pdf` file online.