UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 2
Fall 2014

Issued: Thursday, September 04, 2014 **Due:** Thursday, September 11, 2014

Problem 1. Consider a binary tree with n levels as shown in Figure 1. Suppose that each link in this tree fails with probability p. Find the probability that there is a working path from the root node (R) to a leaf. You can find a recursive formula for this probability.

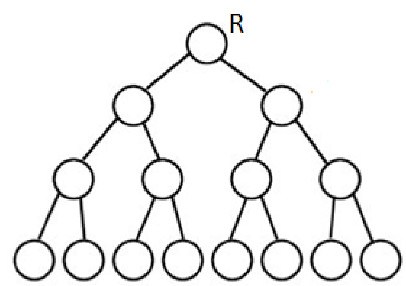

Figure 1: reliability graph for a binary tree with $n = 3$.

Problem 2. Indicate whether the following statements are true. If yes, give a short proof. If no, provide a counterexample.

(a) Disjoint events are independent.

(b) The variance of a sum of random variables is the sum of their variances.

(c) The expected value of a sum of random variables is the sum of their expected values.

Problem 3. Consider a balls-and-bins model with K balls and M bins as shown in Figure 2. Each ball is thrown to exactly d bins randomly. For example, in the figure, ball 1 is in bins 1 and 2, and $d = 2$. The balls-and-bins model can also be viewed as a random bipartite graph[1]. Thus, the degree of a ball (number of bins that the ball is thrown in) is fixed. However, the degree of the bins (number of balls thrown in a bin) is a random variable.

[1] A bipartite graph is a graph whose vertices can be partitioned to two sets such that there are no edges connecting any two vertices of the same set.

(a) Find the distribution of this random variable.

(b) What is the expected degree of a bin?

(c) Now suppose you pick a random edge in the graph. What is the distribution of the degree of the right node connected to this edge? Is it the same as part (a)?

(d) We call a bin with degree 1 a *singleton*. What is the average number of singletons in a random balls-and-bins model?

(e) Find the average number of balls that are connected to at least one singleton.

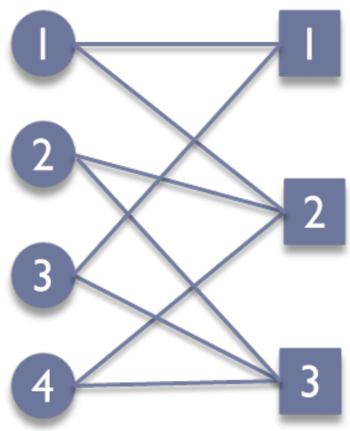

Figure 2: Balls-and-bins model with $K = 4$, $M = 3$ and $d = 2$.

Problem 4. You are handed two envelopes, and you know that each contains a positive integer dollar amount and that the two amounts are different. The values of these two amounts are modeled as constants which are unknown. Without knowing what the amounts are, you select an envelope randomly, and after looking at the amount inside, you might switch the envelopes. A friend claims that the following strategy will increase above $1/2$ the probability of ending up with the larger amount: toss a coin repeatedly; let X be equal to $1/2$ plus the number of tosses required to obtain heads for the first time, and switch if the amount in the envelope you selected is less than the value of X. Is your friend correct?

Problem 5. Consider a biased coin with p being the probability of heads. We flip the coin until r tails have appeared, and then stop flipping the coin. Let X be the random variable denoting the number of heads in this experiment.

(a) Find the distribution (PMF) of X.

(b) Find the expected value of X.

(c) Find the variance of X.

Mini-Lab. Download `Lab2 - Cloud Data Insurance.ipynb` from bCourses. Complete the mini-lab by filling missing code blocks, and working on problems. Submit your `ipynb` file and `pdf` file online.