UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 3
Fall 2014

Issued: Thursday, September 11, 2014 **Due:** Thursday, September 18, 2014

Problem 1. Jim and George are setting up venture capital portfolios. Suppose that Jim picks $n+1$ startups to fund and George picks n startups to fund. Suppose that the probability of any startup succeeding is 50% and all of the startups succeed or fail independently. Show that the probability that Jim picks more winners than George is 1/2.

Problem 2. Suppose that the number of people that come for shopping to a mall has a Poisson distribution with parameter λ. Assume that each shopper is a male with probability p and a female with probability $1-p$ independent of everything else. Find the PMF of male shoppers.

Problem 3. A needle of length ℓ is dropped randomly on a plane surface that is partitioned in rectangles by horizontal lines that are a apart and by vertical lines that are b apart. Suppose that $\ell < a$ and $\ell < b$. What is the expected number of rectangle sides crossed by the needle? What is the probability that the needle crosses at least one side of a rectangle?

Problem 4. Two points are picked uniformly at random in the interval $[0, L]$.

(a) What is the expected distance between these points?

(b) Suppose that the selected points are X_1 and X_2 such that $0 \leq X_1 \leq X_2 \leq L$. What is the probability that a triangle can be formed from the lengths X_1, $X_2 - X_1$ and $L - X_2$?

Problem 5. Suppose that the continuous random variable X has p.d.f.

$$f(x) = c(1-x^2)1_{\{-1 \leq x \leq 1\}},$$

where $1_A = 1$ if A is true and $1_A = 0$ if A is not true.

(a) Find c such that the p.d.f. is valid.

(b) Find the expected value of X.

(c) Find the variance of X.

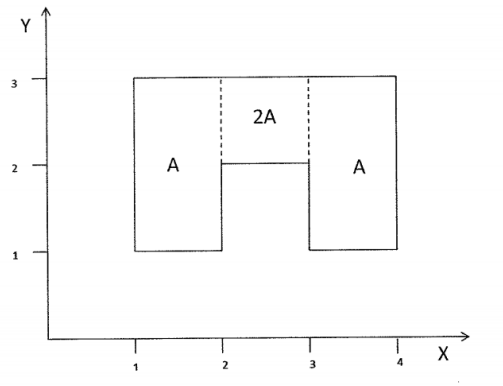

Figure 1: Joint pdf of X and Y.

Problem 6. Figure 1 shows the joint density $f_{X,Y}$ of random variables X and Y.

(a) Find A and sketch f_X, f_Y and $f_{X|X+Y\leq 3}$.

(b) Find $E[X|Y=y]$ for $1 \leq y \leq 3$ and $E[Y|X=x]$ for $1 \leq x \leq 4$.

(c) Find $cov(X,Y)$.

Mini-Lab. Download `Lab3 - Multimedia Part I.ipynb` from course websites. Complete the mini-lab by filling missing code blocks, and working on problems. Submit your `ipynb` file and `pdf` file online.