UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 5
Fall 2014

Issued: Thursday, October 2, 2014 **Due:** Thursday, October 9, 2014

Problem 1. Let $X_1, X_2, \ldots, X_n$ be n i.i.d. exponential random variables with mean 1.

(a) Find the pdf of $Y = X_1 + X_2 + \ldots + X_n$.

(b) Find the pdf of $Z = \frac{X_1}{Y}$.

(c) Are Y and Z independent?

(d) Let $X_M = \max_i X_i$ be the maximum of these random variables. What is the joint distribution of X_M and the rest of the random variables?

Problem 2. In order to estimate the probability of head in a coin flip, p, you flip a coin n times, and count the number of heads, S_n. You use the estimator $\hat{p} = S_n/n$. You choose the sample size n to have a guarantee

$$\Pr(|S_n/n - p| \geq \epsilon) \leq \delta.$$

Determine how the value of n suggested by Chebyshev inequality changes when ϵ is reduced to half of its original value? How does it change when δ is reduced to half of its original value?

Problem 3. Before starting to play roulette in a casino, you want to look for biases that you can exploit. You therefore watch 100 rounds that result in a number between 1 and 36, and count the number of odd results. If the count exceeds 55, you decide that the roulette is not fair. Assuming that the roulette is fair, what is the approximate probability that you will make the wrong decision.

Problem 4. Using Central Limit Theorem, compute the number of people to poll in a public opinion survey to estimate the fraction of the population that will vote in favor of a proposition within α percent, with probability at least $1 - \beta$.

Problem 5. Let X_i, $1 \leq i \leq n$ be a sequence of i.i.d. random variables distributed uniformly in $(-1, 1)$. Show that the following sequences converge in probability to some limit.

- $Y_n = X_n/n$.
- $Y_n = (X_n)^n$.
- $Y_n = X_1 X_2 \ldots X_n$.
- $Y_n = \max\{X_1, X_2, \ldots, X_n\}$.

Problem 6. Consider the Markov chain of Figure 1 where a and b are in $(0, 1)$.

- Find the invariant distribution.
- Calculate $\Pr(X(1) = 1, X(2) = 0.X(3) = 0, X(4) = 1|X(0) = 0)$.
- Show that the Markov chain is aperiodic.

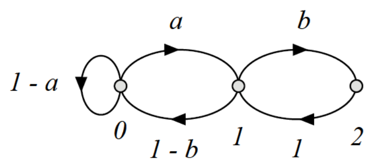

Figure 1: A simple Markov Chain

Problem 7. You have a database of an infinite number of movies. Each movie has a rating that is uniformly distributed in $[0, 5]$ independent of all other movies. You choose movies from the database one by one until you find two such that the sum of their ratings exceeds 7.5. What is the expected number of movies you will have to choose?

Mini-Lab. Download `Lab5 - BlueBox.ipynb` from course websites. Complete the mini-lab by filling missing code blocks, and working on problems. Submit your `ipynb` file and `pdf` file online.