EE126: Probability and Random Processes

Problem Set 6

Fall 2014

Issued: Thursday, October 9, 2014   **Due:** Thursday, October 16, 2014

Problem 1. Draw the smallest irreducible periodic Markov Chain. Show that the fraction of time in the states converges but the probability of being in a state at time n does not converge.

Problem 2. Consider the following Markov chain. Suppose that $X(0) = 1$. Calculate the average number of times it visits state 1 before being absorbed in state 3.
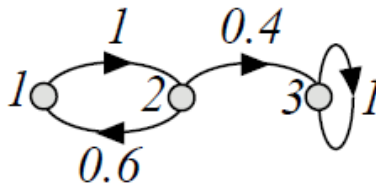

Figure 1: Markov Chain

Problem 3. A man tries to go up a ladder than has N rungs. Every step he makes, he has a probability p of dropping back to the ground and he goes up one rung otherwise. Calculate the average time he takes to reach the top.

Problem 4. Let $\{X_n, n \geq 0\}$ be a Markov chain with two states, -1 and 1, and $P(-1, 1) = P(1, -1) = a$ for $a \in (0, 1)$. Define,

$$Y_n = X_0 + X_1 + \ldots + X_n.$$

Is $\{Y_n, n \geq 0\}$ a Markov chain? Prove or disprove.

Problem 5. Consider a Poisson process $\{N_t, t \geq 0\}$ with rate $\lambda = 1$. Let random variable S_i denote the time of the i-th arrival.

(a) Given $S_3 = s$, find the joint distribution of S_1 and S_2. Show you work.

(b) Find $E[S_2|S_3 = s]$.

(c) Find $E[S_3|N_1 = 2]$.

Problem 6. Consider two independent Poisson processes with rates λ_1 and λ_2. Those processes measure the number of customers arriving in store 1 and 2.

(a) What is the probability that a customer arrives in store 1 before any arrives in store 2?

(b) What is the probability that in the first hour exactly 6 customers arrive at the two stores? (The total for both is 6)

(c) Given exactly 6 have arrived at the two stores, what is the probability all 6 went to store 1?

Mini-Lab. Download `Lab6 - PageRank.ipynb` from course websites. Complete the mini-lab by filling missing code blocks, and working on problems. Submit your `ipynb` file and `pdf` file online.