UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: PROBABILITY AND RANDOM PROCESSES

Problem Set 8
Fall 2014

Issued: Thursday, October 23, 2014 **Due:** Thursday, October 30, 2014

Problem 1. Given $X = i$, $Y = Exp(\lambda_i)$, for $i = 0, 1$. Assume $\lambda_0 < \lambda_1$. Find $\hat{X} = MAP[X|Y]$ and calculate $\Pr(\hat{X} \neq X)$. We know $\pi(i) = \Pr(X = i)$ for $i = 0, 1$.

Problem 2. Consider random variable Y which is uniform in $[a, b]$. We know that $a \in \{a_1, a_2, \ldots, a_M\}$ and $b \in \{b_1, b_2, \ldots, b_M\}$ for some integer $M > 1$. You observe n i.i.d. samples of this random variable $y_1, \ldots, y_n$. Calculate the Maximum-likelihood estimator of a and b.

Problem 3. Consider the following communication channel. There is an i.i.d. source that generates symbols $\{1, 2, 3, 4\}$ according to a prior distribution $\pi = [p_1, p_2, p_3, p_4]$. The symbols are modulated by QPSK scheme, i.e. they are mapped to constellation points $(\pm 1, \pm 1)$. The communication is on a baseband Gaussian channel, i.e. if the sent signal is $(x_1, x_2),\ x_i \in \{-1, 1\}$, the received signal is

$$y_1 = x_1 + z_1,$$

$$y_2 = x_2 + z_2,$$

where z_1 and z_2 are independent $N(0, \sigma^2)$ random variables. Find the MAP detector of this communication channel.

Problem 4. Consider a random bipartite graph, G_1, with K left nodes and M right nodes. Each of the KM possible edges of this graph is connected with probability p independently. (The model is similar to sample midterm 1, problem 3.)

(a) As M and K get large, how many left nodes are connected to right nodes of degree 1 or singletons?

(b) A doubleton is a right node of degree two. As M and K get large, how many doubletons do we have?

(c) We call 2 doubletons distinct, if they are not connected to the same 2 left nodes. As K and M get large, what is the probability that two doubletons are distinct?

Problem 5. Consider the same setting as the previous problem.

(d) Let M_s be the number of doubletons for which both of the left nodes are also connected to singletons. Find M_s as K and M get large.

(e) We construct another random graph, G_2, as follows. Let K_s be the number of left nodes that are connected to singletons, which you calculated in part (a). Graph G_2 has K_s nodes corresponding to these left nodes. Two nodes in G_2 are connected if there is a doubleton in G_1 that is connected to those left nodes. Thus, G_2 has M_s edges which you calculated in part (d). Argue that G_2 is equivalent to an Erdos-Renyi random graph.

(f) An Erdos-Renyi random graph $G(N, q)$ has a giant component of size linear in N if $Nq > 1$. A giant component is the largest set of nodes in the graph that is connected. Suppose that $M = 4K$. Find a condition on p as a function of K such that G_2 has a giant component.

Mini-Lab. Download `Lab8 - Distributed Storage Systems.ipynb` from course websites. Complete the mini-lab by filling missing code blocks, and working on problems. Submit your `ipynb` file and `pdf` file online.