UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 9
Fall 2014

Issued: Thursday, October 30, 2014 **Due:** Thursday, November 6, 2014

Problem 1. The random variable X is exponentially distributed with mean 1. Given X, the random variable Y is exponentially distributed with rate X (with mean $1/X$).

(a) Find $MLE[X|Y]$;

(b) Find $MAP[X|Y]$.

Problem 2. Consider random variable Y which is exponentially distributed with parameter θ. You observe n i.i.d. samples of this random variable $y_1, \ldots, y_n$. Calculate the Maximum-likelihood estimator of θ. Let $\theta_{ML}(y_1, \ldots, y_n)$ be the ML estimator. What is the bias of this estimator?

Problem 3. You are testing a digital link that corresponds to a BSC with some error probability $\epsilon \in [0, 0.5)$.

(a) Assume you observe the input and the output of the link. How do you find the MLE of ϵ?

(b) You are told that the inputs are i.i.d. bits that are equal to 1 with probability 0.6 and to 0 with probability 0.4. You observe n outputs. How do you calculate the MLE of ϵ.

(c) The situation is as in the previous case, but you are told that ϵ has pdf $4 - 8x$ on $[0, 0.5)$. How do you calculate the MAP of ϵ given n outputs.

Problem 4. Consider a hypothesis testing problem that if $X = 0$, you observe a sample of $N(\mu_0, \sigma^2)$, and if $X = 1$, you observe a sample of $N(\mu_1, \sigma^2)$. Find the Neyman-Pearson test for false alarm α, i.e. $\Pr(\hat{X} = 1|X = 0) \leq \alpha$.

Problem 5. The situation is the same as in the previous problem. You observe n inputs and outputs of the BSC. You want to solve a hypothesis problem to detect that $\epsilon > 0.1$ with a probability of false alarm at most equal to 0.05. Assume that n is very large and use the CLT.

Problem 6. If $X = 0$, $Y = U[-1, 1]$ and if $X = 1$, $Y = U[0, 2]$. Solve a hypothesis testing problem so that the probability of false alarm is less than or equal β.

Mini-Lab. Download `Lab9 - Random Graphs and Matrix Completion.ipynb` from course websites. Complete the mini-lab by filling missing code blocks, and working on problems. Submit your `ipynb` file and `pdf` file online.