UC Berkeley
Department of Electrical Engineering and Computer Sciences
EE126: Probability and Random Processes

**Discussion 3**

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*Problem* 1. Let $X \sim \text{Unif}[0,1]$. If $Y = 2X$, what is the PDF of $Y$?

*Problem* 2. Let $X \sim \text{Unif}[-1,1]$. Let $Y = \text{abs}(X)$. Find the PDF of $Y$.

*Problem* 3. Let $X, Y \sim \text{Unif}[0,1]$ and $Z = X + Y$. Find the PDF of $Z$.

*Problem* 4. An S school grad student goes to a casino in Las Vegas, and finds a newly launched game called "Baskin-Robbins 31". The rule of BR 31 is as follows: player and dealer independently draw their own random integer, uniformly distributed between 1 and 100. If the absolute difference between the two numbers is less than 31, the dealer takes the pot, and the player wins the pot otherwise. The S school grad student quickly claims that the game is wrongly designed and indeed favors players. Is he correct? If not, why?

*Problem* 5. Let $X_1, X_2, \ldots, X_n$ be i.i.d. continuous random variables distributed uniformly between 0 and 1.

(a) Find $E(X_i)$ and $var(X_i)$.

(b) Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the ordered random variables such that $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$. Find the distributions of $X_{(1)}$ and $X_{(n)}$.

(c) Find the expected value of $X_{(1)}$ and $X_{(n)}$.

(d) Assume that $n = 7$. Find the distribution of $X_{(5)} - X_{(4)}$.

*Problem* 6 (Skipped, but easy.). Reading time of a data block from a hard disk $T$ can be approximated as a sum of seek time and transfer time, i.e.,

$$T = T_{\text{seek}} + T_{\text{rotation}} + T_{\text{control overhead}} + T_{\text{transfer}},$$

where seek time $T_{\text{seek}} = a \times$ seek distance, rotation time $T_{\text{rotation}} = b \times$ rotation angle, control overhead time $T_{\text{control overhead}} = c$, and transfer time $T_{\text{transfer time}} = d \times$ block size.

a) Assume random I/Os. That is, each I/O request reads (or writes) a data block that is located at a (track, sector) chosen uniformly at random from $[0,1] \times [0, 2\pi]$. What is the distribution of seek distance (radial distance)? What is the expected value?

b) Further assume that cylinders of a disk can rotate only one direction. What is the distribution of rotation angle?

c) Further assume that file size is exponentially distributed with parameter $\mu$. What is the distribution of $T_{\text{transfer}}$? What is the mean? What is the variance?

d) How can we find the distribution of $T$? What is the mean? What is the variance?