UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: PROBABILITY AND RANDOM PROCESSES

Discussion 5
Fall 2015

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1 Some Quick Notes

(1) For a random variable X with transform $M_X(s)$, the transform of $Y = aX + b$ is $M_Y(s) = e^{bs}M_X(as)$.

(2) Transforms of a random variable may not exist for any value of s. Therefore we should specify the range in which s can take value. Examples are the geometric random variables and the exponential random variables.

(3) The Chebyshev inequality is a two-sided inequality, meaning that it is bounding the probability that X is far from $\mathrm{E}[X]$ on both sides. However, there is also one-sided Chebyshev inequality. Please search for Cantelli's inequality if you want to learn more.

2 Problems

Problem 1. (a) Suppose the transform for L is:

$$M_L(s) = \left(\frac{1}{3} + \frac{2}{3}e^s\right)^{10} e^{2s}$$

Determine the PMF $p_L(l)$.

(b) Suppose the transform for K is:

$$M_K(s) = \frac{\frac{1}{5}e^{4s}}{1 - \frac{4}{5}e^s}, \quad s < \ln(\frac{5}{4})$$

Determine $p_K(k)$, and evaluate $\mathrm{E}[K]$ and σ_K^2.

Problem 2. Many casino games are only slightly biased in favor of the casino, so that the casino makes a profit while customers maintain interest. Imagine such a game, where the probability of the casino winning is 0.51. Suppose you play 400 games, and let L denote the number of times you lose. Use whichever approximations to the binomial you feel are appropriate to calculate the following:

(a) $P(190 \le L \le 210)$

(b) $P(210 \leq L \leq 230)$

Problem 3. Let X be the sum of 20 i.i.d. Poisson random variables $X_1, \ldots, X_{20}$ with $\mathrm{E}[X_i] = 1$. Use Markov's inequality, Chebyshev's inequality, and Chernoff Bound to find an upper bound of $P(X \geq 26)$. Use CLT to estimate $P(X \geq 26)$.

Problem 4. A brief introduction to Markov chains.