UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion 7
GSI: Kangwook Lee

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Problem 1 (SP06 Final P4). Consider a Markov chain with three states 0, 1, and 2. The transition probabilities are $P(0,1) = P(0,2) = \frac{1}{2}$, $P(1,0) = P(1,1) = \frac{1}{2}$, and $P(2,0) = \frac{2}{3}$, $P(2,2) = \frac{1}{3}$.

(a) Classify the states in the chain. Is this chain periodic or aperiodic?

(b) In the long run, what fraction of time does the chain spend in state 1?

(c) Suppose that X_0 is chosen according to the steady state distribution. $P(X_0 = 0|X_2 = 2)$?

(d) Suppose that $X_0 = 0$, and let T denote the first time by which the process has visited all the states. Find $E[T]$.

Problem 2 (FA06 Final P4). At the nuclear power plant, one of the reactors begins to melt down. The emission of radioactive particles can be modeled as a Poisson process with rate $\lambda = 100$ particles/second.

(a) What is the expected total number of particles that escape the time window $[10, 20] \cup [50, 60]$ seconds?

(b) Suppose that exactly 300 particles escaped in the first second. What is the PDF of the number of particles that escape in the next second?

In order to stop the spread of radiation, the plant is equipped with a set of n shields that are either ON or OFF. Any OFF shield has no effect on the particle stream, whereas any ON shield blocks each particle with probability p, and let it through with probability $1 - p$, independently of all other particles. Suppose that each shield acts independently of all the other shields.

(c) For some fixed integer k (with $1 \leq k \leq n$), suppose that exactly k of the shields are ON, and consider the stochastic process defined by the particles that end up escaping. Prove that the expected number of particles that escape per second is equal to $100(1 - p)^k$. Is this a Poisson process?

(d) Now suppose that each shield is turned ON (independently of all other shields) with probability 0.50. Let X_n represent the number of particles that escape in the first second. (Recall that fixed integer n is the total number of shields.) What is $E[X_n]$? What is $\text{var}(X_n)$?

(e) Does the sequence $\{X_n\}$ converge in probability?

(f) Is this new sequence a Poisson process?