

**Problem Set 1**

Fall 2015

**Issued:** Thursday, August 27, 2015

**Due:** Thursday, September 03, 2014

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*Problem 1.* Find an example of 3 events  $A$ ,  $B$ , and  $C$  such that each pair of them are independent, but they are not mutually independent. Show the calculations.

*Problem 2.* Alice and Bob want to choose between the opera and the movies by tossing a fair coin. Unfortunately, the only available coin is biased (though the bias is not known exactly). How can they use the biased coin to make a decision so that either option is equally likely?

*Problem 3.* Show that if  $\Pr(A|B) > \Pr(A|\bar{B})$ , then  $\Pr(B|A) > \Pr(B|\bar{A})$ . Here  $\bar{A}$  denotes the complementary event of  $A$ .

*Problem 4.* Alice has a high fever and go to the doctor to identify the cause. 3% of the people have H1N1, 20% of the people have the flu, and the rest have neither. Suppose that 100% of the H1N1 people have a high fever, 50% of the flu people have a high fever, and 2% of the people who have neither, have a high fever. Is it more likely that Alice has H1N1, the flu, or neither?

*Problem 5.* There are  $m$  people passing a ball to each other. At the beginning, Bob has the ball. In each round, the person who has the ball passes this ball equally likely to one of the other  $m - 1$  people. What is the probability that in the  $n$ th round, the ball is passed from Bob?

*Problem 6.* Figure 1 is the reliability graph of a system. The links of the graph represents components of the system. Each link is working with probability  $p$  and defective with probability  $1 - p$ , independently of the other links. The system is operational if the nodes  $S$  and  $T$  are connected. Calculate the probability that the system is operational.

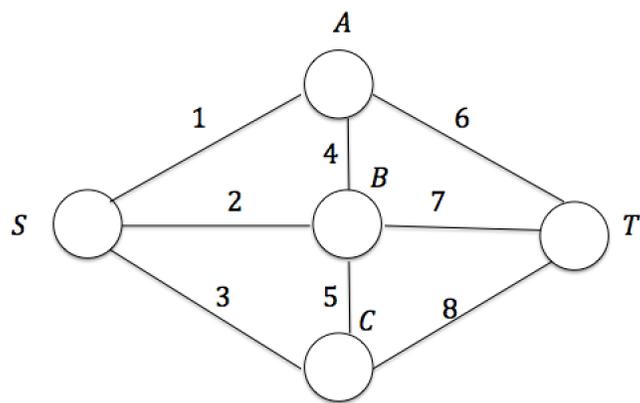


Figure 1: reliability graph