UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 4
Fall 2015

Issued: Thursday, September 24, 2015 **Due:** 9:00am Thursday, October 1, 2015

Problem 1. Midterm 01.

Problem 2. Suppose we have two independent Gaussian random variables $X \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $Y \sim \mathcal{N}(\mu_2, \sigma_2^2)$. Define random variables Z and W as mixtures as X and Y. More specifically, let U and V be independent Bernoulli random variables, with $\Pr(U = 1) = p$ and $\Pr(V = 1) = q$, and $Z = X\mathbf{1}\{U = 1\} + Y\mathbf{1}\{U = 0\}$, $W = X\mathbf{1}\{V = 1\} + Y\mathbf{1}\{V = 0\}$, where $\mathbf{1}$ denotes indicator function.

(a) Find $\mathbb{E}[Z]$ and $\text{var}(Z)$.

(b) Find $\text{cov}(Z, W)$.

Problem 3. Let X and Y be two independent standard normal random variables, $\mathcal{N}(0, 1)$. Let $W = X^2 + Y^2$ and $Z = X/Y$. Find the marginal distributions of Z and W, and show that they are independent.

Problem 4. Let X, Y, and Z be independent random variables. X is Bernoulli with $p = 1/4$. Y is exponential with parameter 3. Z is Poisson with parameter 5.

(a) Find the transform of $5Z + 1$.

(b) Find the transform of $X + Y$.

(c) Consider the new random variable $U = XY + (1 - X)Z$. Find the transform associated with U.