UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 7

Fall 2015

Issued: Thursday, October 15, 2015 **Due:** 9:00am, Thursday, October 22, 2015

Problem 1. Consider a Poisson process $\{N_t, t \geq 0\}$ with rate $\lambda = 1$. Let random variable S_i denote the time of the i-th arrival.

(a) Given $S_3 = s$, find the joint distribution of S_1 and S_2.

(b) Find $\mathbb{E}[S_2 | S_3 = s]$.

(c) Find $\mathbb{E}[S_3 | N_1 = 2]$.

Problem 2. Each morning, as you pull out of your driveway, you would like to make a U-turn rather than drive around the block. Unfortunately, U-turns are illegal and police cars drive by according to a Poisson process with rate λ. You decide to make a U-turn one you see that the road has been clear of police cars for τ units of time. Let N be the number of police cars you see before you make a U-turn.

(a) Find $\mathbb{E}[N]$.

(b) Find the conditional expectation of the time elapsed between police cars $n-1$ and n, given that $N \geq n$.

(c) Find the expected time that you wait until you make a U-turn.

Problem 3. A queue has Poisson arrivals with rate λ. It has two servers that work in parallel. Where there are at least two customers in the queue, two are being served. When there is only one customer, only one server is active. The service times are i.i.d. Exp(μ).

(a) Argue that the queue length is a Markov Chain.

(b) Draw the state transition diagram.

(c) Find the minimum value of μ so that the queue is positive recurrent and solve the balance equations.