UC Berkeley
Department of Electrical Engineering and Computer Sciences

ELECTRICAL ENGINEERING 126: PROBABILITY AND RANDOM PROCESSES

Discussion 10
Fall 2017

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1. **Statistical Estimation**

   Given $X \in \{0, 1\}$, the random variable Y is exponentially distributed with rate $3X + 1$.

   (a) Assume $\mathbb{P}(X = 1) = p \in (0, 1)$ and $\mathbb{P}(X = 0) = 1 - p$. Find the MAP estimate of X given Y.

   (b) Find the MLE of X given Y.

   (c) Solve the hypothesis testing problem of X given Y with a probability of false alarm at most 0.1. That is, find $\hat{X}$ as a function of Y that maximizes $\mathbb{P}(\hat{X} = 1 \mid X = 1)$ subject to $\mathbb{P}(\hat{X} = 1 \mid X = 0) \le 0.1$.

   (d) For what value of p does one have the same solution for (a) and (c)?

2. **Exponential MLE, MAP, Hypothesis Testing**

   The random variable X is exponentially distributed with mean 1. Given X, the random variable Y is exponentially distributed with rate X.

   (a) Find MLE$[X \mid Y]$;

   (b) Find MAP$[X \mid Y]$;

   (c) Solve the following hypothesis testing problem:

   Maximize $\mathbb{P}(\hat{X} = 1 \mid X = 1)$ subject to $\mathbb{P}(\hat{X} = 1 \mid X = a) \le 5\%$

   where $a > 1$ is given.

3. **Laplace Prior & ℓ^1-Regularization**

   Suppose you draw n i.i.d. data points $(x_1, y_1), \ldots, (x_n, y_n)$, where n is a positive integer and the true relationship is $Y = WX + \varepsilon$, $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. (That is, Y has a linear dependence on X, with additive Gaussian noise.) Further suppose that W has a prior distribution with density

   $$f_W(w) = \frac{1}{2\beta} \mathrm{e}^{-|w|/\beta}, \qquad \beta > 0.$$

   (This is known as the **Laplace distribution**.) Show that finding the MAP estimate of W given the data points $\{(x_i, y_i) : i = 1, \ldots, n\}$ is equivalent to minimizing the cost function

   $$J(w) = \sum_{i=1}^{n} (y_i - wx_i)^2 + \lambda |w|$$

   (you should determine what λ is). This is interpreted as a one-dimensional ℓ^1-regularized least-squares criterion, also known as LASSO.