UC Berkeley
Department of Electrical Engineering and Computer Sciences

Electrical Engineering 126: Probability and Random Processes

Discussion 14

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1. **Frogs**

   Three frogs are playing near a pond. When they are in the sun they get too hot and jump in the lake at rate 1. When they are in the lake they get too cold and jump onto the land at rate 2. The rates here refer to the rate in exponential distribution. Let X_t be the number of frogs in the sun at time $t \geq 0$.

   (a) Find the stationary distribution for $(X_t)_{t \geq 0}$.

   (b) Check the answer to (a) by noting that the three frogs are independent two-state Markov chains.

2. **Lazy Server**

   Customers arrive at a service facility at the times of a Poisson process of rate λ. The service facility has infinite capacity. There is an infinitely powerful but lazy server who visits the service facility at the times of a Poisson process of rate μ. The Poisson process of server visits is independent of the Poisson process of arrival times of the customers. When the server visits the facility she instantaneously serves all the customers that are currently waiting in the facility and then immediately leaves (until her next visit).

   Thus, for instance, at any time, any customers that are waiting in the service facility would only be those that arrived after the most recent visit of the server.

   (a) Show that the system admits a well-defined stationary regime for all values of the parameters $\lambda > 0$ and $\mu > 0$.

   (b) Find the mean number of customers waiting in the system at any given time in the stationary regime.

3. **Queueing MDP**

   Consider a queue with Poisson arrivals with rate λ. The queue can hold N customers (N is a positive integer). The service times are i.i.d. Exponential(μ). When a customer arrives, you can choose to pay him $c > 0$ so that he does not join the queue. You also pay c when a customer arrives at a full queue. You want to decide when to accept customers to minimize the cost of rejecting them, plus the cost of the average waiting time they spend in the queue.

   Formulate the problem as a Markov decision problem. For simplicity, consider a total discounted cost. That is, if x_t customers are in the system at time

$t \geq 0$, then the waiting cost during $[t, t+\epsilon]$ is $\mathrm{e}^{-\beta t} x_t \epsilon$, for $\epsilon > 0$. Similarly, if you reject a customer at time t, then the cost is $c\mathrm{e}^{-\beta t}$. Write the dynamic programming equations.