

Discussion 3

Fall 2017

1. Order Statistics

For n a positive integer, let X_1, \dots, X_n be i.i.d. random variables with common density function f and CDF F . For $i = 1, \dots, n$, let $X^{(i)}$ be the i th smallest of X_1, \dots, X_n , so we have $X^{(1)} \leq \dots \leq X^{(n)}$. $X^{(i)}$ is known as the **i th order statistic**.

- (a) What is the CDF of $X^{(i)}$?
- (b) Differentiate the CDF to obtain the density of $X^{(i)}$.
- (c) Can you obtain the density of $X^{(i)}$ directly?

2. Change of Variables

- (a) Suppose that X has the **standard normal distribution**, that is, X is a continuous random variable with density function

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right).$$

What is the density function of $\exp X$? (The answer is called the **log-normal distribution**.)

- (b) Suppose that X is a continuous random variable with density f . What is the density of X^2 ?
- (c) What is the answer to the previous question when X has the standard normal distribution? (This is known as the **chi-squared distribution**.)

3. Expected Norm

Pick two points X and Y independently and uniformly in $[0, 1]^2$. Calculate $\mathbb{E}[\|X - Y\|_2^2]$.