UC Berkeley
Department of Electrical Engineering and Computer Sciences

Electrical Engineering 126: Probability and Random Processes

Discussion 6

Fall 2017

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1. **Poisson Practice**

   Let $(N(t), t \geq 0)$ be a Poisson process with rate λ. Let T_k denote the time of k-th arrival, for $k \in \mathbb{N}$, and given $0 \leq s < t$, we write $N(s,t) = N(t) - N(s)$. Compute:

   (a) $\mathbb{P}(N(1) + N(2,4) + N(3,5) = 0)$.

   (b) $\mathbb{E}(N(1,3) \mid N(1,2) = 3)$.

   (c) $\mathbb{E}(T_2 \mid N(2) = 1)$.

2. **Customers in a Store**

   Consider two independent Poisson processes with rates λ_1 and λ_2. Those processes measure the number of customers arriving in store 1 and 2.

   (a) What is the probability that a customer arrives in store 1 before any arrives in store 2?

   (b) What is the probability that in the first hour exactly 6 customers arrive at the two stores? (The total for both is 6.)

   (c) Given exactly 6 have arrived at the two stores, what is the probability all 6 went to store 1?

3. **Sum-Quota Sampling**

   Consider the problem of estimating the mean interarrival time of a Poisson process. In what follows, recall that N_t denotes the number of arrivals by time t, where $t \geq 0$.

   Sum-quota sampling is a form of sampling in which the number of samples is not fixed in advance; instead, we wait until a fixed *time* t, and take the average of the interarrival times seen so far. If we let X_i denote the ith interarrival time, then

   $$\bar{X} = \frac{X_1 + \cdots + X_{N_t}}{N_t}.$$

   Of course, the above quantity is not defined when $N_t = 0$, so instead we must condition on the event $\{N_t > 0\}$. Compute $\mathbb{E}[\bar{X} \mid N_t > 0]$, assuming that $(N_t,\ t \geq 0)$ is a Poisson process of rate λ.