UC Berkeley
Department of Electrical Engineering and Computer Sciences

ELECTRICAL ENGINEERING 126: PROBABILITY AND RANDOM PROCESSES

Discussion 9
Fall 2017

---

1. **Estimating an Exponential Distribution**

   (a) You draw a sample $X_1, \ldots, X_n$ (n is a positive integer) for the lifetime of a light bulb (assumed to be exponentially distributed). You have information from a trustworthy source that the rate of the exponential distribution satisfies $\lambda \geq 2$. Using Chebyshev's Inequality, what is the minimum n required to construct a confidence interval for the mean lifetime of the light bulb? Your confidence interval must have tolerance at most ε with confidence at least $1 - \delta$ for parameters $\delta, \varepsilon > 0$. Also, you should precisely state what your estimate for the mean lifetime is.

   (b) Due to budget constraints, you are only allowed to use 10000 samples. You must still maintain the ε tolerance, but with such a large sample size, you feel justified in using the Central Limit Theorem. What is your new confidence? (Again, you may use the information from your trustworthy source. Express your answers in terms of Φ, the CDF of the standard normal distribution.)

2. **Random Telegraph Wave**

   Let $\{N_t,\ t \geq 0\}$ be a Poisson process with rate λ and define $X_t = X_0(-1)^{N_t}$ where $X_0 \in \{0, 1\}$ is a random variable independent of N_t.

   (a) Does the process X_t have independent increments?

   (b) Calculate $\mathbb{P}(X_t = 1)$ if $\mathbb{P}(X_0 = 1) = p$.

   (c) Assume that $p = 0.5$. Calculate $\mathbb{E}[X_{t+s}X_s]$ for $s, t \geq 0$.

3. **Markov Chains Meet Linear Algebra**

   Consider the transition matrix:

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

   (a) Find P^n, for each positive integer n.
   Hint: This can be done without any math.

   (b) Find the distinct eigenvalues of P along with their multiplicities.

   (c) Can you write $P = U\Lambda U^{-1}$ for some diagonal matrix Λ and invertible matrix U?