UC Berkeley
Department of Electrical Engineering and Computer Sciences

Electrical Engineering 126: Probability and Random Processes

Problem Set 5

Fall 2017

Issued: September 28, 2017 **Due:** 9 AM, Thursday, October 5, 2017

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1. **Basketball**

   Michael misses shots with probability 1/4, independently of other shots.

   (a) What is the expected number of shots that Michael will make before he misses three times?

   (b) What is the probability that the second and third time Michael makes a shot will occur when he takes his eigth and ninth shots, respectively?

   (c) What is the probability that Michael misses two shots in a row before he makes two shots in a row?

2. **Bus Arrivals at Cory Hall**

   Starting at time 0, the F line makes stops at Cory Hall according to a Poisson process of rate λ. Students arrive at the stop according to an independent Poisson process of rate μ. Every time the bus arrives, all students waiting get on.

   (a) Given that the interarrival time between bus $i-1$ and bus i is x, where i is a positive integer ≥ 2, find the distribution for the number of students entering the ith bus.

   (b) Given that a bus arrived at 9:30 AM, find the distribution for the number of students that will get on the next bus.

   (c) Find the distribution of the number of students getting on the next bus to arrive after 11:00 AM. (You can assume that time 0 was infinitely far in the past.)

3. **Poisson Process Warm-Up**

   Consider a Poisson process $\{N_t, t \geq 0\}$ with rate $\lambda = 1$. Let random variable S_i denote the time of the ith arrival, where i is a positive integer.

   (a) Given $S_3 = s$, where $s > 0$, find the joint distribution of S_1 and S_2.

   (b) Find $\mathbb{E}[S_2 \mid S_3 = s]$.

   (c) Find $\mathbb{E}[S_3 \mid N_1 = 2]$.

   (d) Give an interpretation, in terms of a Poisson process with rate λ, of the following fact:

If N is a geometric random variable with parameter p, and $(X_i)_{i\in\mathbb{N}}$ are i.i.d. exponential random variables with parameter λ, then $X_1 + \cdots + X_N$ has the exponential distribution with parameter λp.

4. Illegal U-Turns

Each morning, as you pull out of your driveway, you would like to make a U-turn rather than drive around the block. Unfortunately, U-turns are illegal and police cars drive by according to a Poisson process with rate λ. You decide to make a U-turn once you see that the road has been clear of police cars for $\tau > 0$ units of time. Let N be the number of police cars you see before you make a U-turn.

(a) Find $\mathbb{E}[N]$.

(b) Let n be a positive integer ≥ 2. Find the conditional expectation of the time elapsed between police cars $n-1$ and n, given that $N \geq n$.

(c) Find the expected time that you wait until you make a U-turn.

5. Expected Squared Arrival Times

Let $(N(t), t \geq 0)$ be a Poisson process with arrival instants $(T_n, n \in \mathbb{N})$, where $0 < T_1 < T_2 < \cdots$. Find $\mathbb{E}(\sum_{k=1}^{3} T_k^2 \mid N(1) = 3)$.