UC Berkeley
Department of Electrical Engineering and Computer Sciences

Electrical Engineering 126: Probability and Random Processes

Problem Set 6
Fall 2017

Issued: October 5, 2017 **Due:** 9 AM, Thursday, October 12, 2017

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1. **Spatial Poisson Process**

   A two-dimensional Poisson process of rate $\lambda > 0$ is a process of randomly occurring special points in the plane such that (i) for any region of area A the number of special points in that region has a Poisson distribution with mean λA, and (ii) the number of special points in non-overlapping regions is independent. For such a process consider an arbitrary location in the plane and let X denote its distance from its nearest special point (where distance between two points (x_1, y_1) and (x_2, y_2) is defined as $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$). Show that:

   (a) $\mathbb{P}(X > t) = \exp(-\lambda \pi t^2)$ for $t > 0$.

   (b) $\mathbb{E}[X] = \dfrac{1}{2\sqrt{\lambda}}$.

2. **Running Sum of a Markov Chain**

   Let $(X_n)_{n \in \mathbb{N}}$ be a Markov chain with two states, -1 and 1, and transition probabilities $P(-1, 1) = P(1, -1) = a$ for $a \in (0, 1)$. Define

   $$Y_n = X_0 + X_1 + \cdots + X_n.$$

   Is $(Y_n)_{n \in \mathbb{N}}$ a Markov chain? Prove or disprove.

3. **Umbrellas**

   A professor has n umbrellas, for some positive integer n. Every morning, she commutes from her home to her office, and every night she commutes from her office back home. On every commute, if it is raining outside, she takes an umbrella (if there is at least one umbrella at her starting location); otherwise she does not take any umbrellas. Assume that on each commute, it rains with probability $p \in (0, 1)$ independently of all other times. Give the state space and transition probabilities for the Markov chain which corresponds to the number of umbrellas she has at her current location.

4. **Markov Chain Practice**

   Consider a Markov chain with three states 0, 1, and 2. The transition probabilities are $P(0, 1) = P(0, 2) = 1/2$, $P(1, 0) = P(1, 1) = 1/2$, and $P(2, 0) = 2/3$, $P(2, 2) = 1/3$.

   (a) Classify the states in the chain. Is this chain periodic or aperiodic?

(b) In the long run, what fraction of time does the chain spend in state 1?

(c) Suppose that X_0 is chosen according to the steady state distribution. What is $\mathbb{P}(X_0 = 0 \mid X_2 = 2)$?

(d) Suppose that $X_0 = 0$, and let T denote the first time by which the process has visited all the states. Find $\mathbb{E}[T]$.

5. Before Absorption

Consider the Markov chain in Figure 1. Suppose that $X(0) = 1$. Calculate the expected number of times that the chain is in state 1 before being absorbed in state 3. ($X(0) = 1$ is included in this number.)

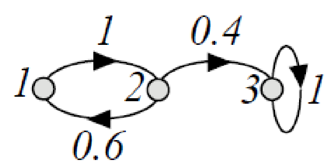

Figure 1: A Markov chain.

6. Random Walk on an Undirected Graph

Consider a random walk on an undirected connected finite graph (that is, define a Markov chain where the state space is the set of vertices of the graph, and at each time step, transition to a vertex chosen uniformly at random out of the neighborhood of the current vertex). What is the stationary distribution?