UC Berkeley
Department of Electrical Engineering and Computer Sciences

Electrical Engineering 126: Probability and Random Processes

Problem Set 9
Fall 2017

Issued: November 2, 2017 **Due:** 9 AM, Thursday, November 9, 2017

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1. **Rotationally Invariant Random Variables**

   You have 2 independent and identically distributed continuous random variables with rotation invariant (isotropic) densities, such that the joint density is also rotation invariant (isotropic). Show that the random variables have the normal distribution.

   [*Hint*: For simplicity, you can take all random variables to be centered, i.e., with zero mean.]

2. **BSC: MLE & MAP**

   You are testing a digital link that corresponds to a BSC with some error probability $\epsilon \in [0, 0.5]$.

   (a) Assume you observe the input and the output of the link. How do you find the MLE of ϵ?

   (b) You are told that the inputs are i.i.d. bits that are equal to 1 with probability 0.6 and to 0 with probability 0.4. You observe n outputs (n is a positive integer). How do you calculate the MLE of ϵ?

   (c) The situation is as in the previous case, but you are told that ϵ has PDF $4 - 8x$ on $[0, 0.5)$. How do you calculate the MAP of ϵ given n outputs?

3. **Poisson Process MAP**

   Customers arrive to a store according to a Poisson process of rate 1. The store manager learns of a rumor that one of the employees is sending 1/2 of the customers to the rival store. Refer to hypothesis $X = 1$ as the rumor being true, that one of the employees is sending every other customer arrival to the rival store and hypothesis $X = 0$ as the rumor being false, where each hypothesis is equally likely. Assume that at time 0, there is a successful sale. After that, the manager observes $S_1, S_2, \ldots, S_n$ where n is a positive integer and S_i is the time of the ith subsequent sale for $i = 1, \ldots, n$. Derive the MAP rule to determine whether the rumor was true or not.

4. **Gaussian LLSE**

   The random variables X, Y, Z are i.i.d. $\mathcal{N}(0, 1)$.

   (a) Find $L[X^2 + Y^2 \mid X + Y]$.

   (b) Find $L[X + 2Y \mid X + 3Y + 4Z]$.

(c) Find $L[(X+Y)^2 \mid X-Y]$.

5. Photodetector LLSE

Consider a photodetector in an optical communications system that counts the number of photons arriving during a certain interval. A user conveys information by switching a photon transmitter on or off. Assume that the probability of the transmitter being on is p. If the transmitter is on, the number of photons transmitted over the interval of interest is a Poisson random variable Θ with mean λ, and if it is off, the number of photons transmitted is 0. Unfortunately, regardless of whether or not the transmitter is on or off, photons may be detected due to "shot noise". The number N of detected shot noise photons is a Poisson random variable N with mean μ. Given the number of detected photons, find the LLSE of the number of transmitted photons.

6. Exam Difficulties

The difficulty of an EE 126 exam, Θ, is uniformly distributed on $[0, 100]$, and Alice gets a score X that is uniformly distributed on $[0, \Theta]$. Alice gets her score back and wants to estimate the difficulty of the exam.

(a) What is the LLSE for Θ?

(b) What is the MAP of Θ?