

# Electrical Engineering 126: Probability & Random Processes

## Midterm 1 Cheat Sheet

Fall 2018

## 1 Distributions

- $X \sim \text{Bernoulli}(p)$ ,  $p \in [0, 1]$ .

PMF:  $p_X(x) = p^x(1-p)^{1-x}$ ,  $x \in \{0, 1\}$ .

MGF:  $M_X(s) = 1 - p + p \exp s$ .

Moments:  $\mathbb{E}[X] = p$ ,  $\text{var } X = p(1-p)$ .

- $X \sim \text{Binomial}(n, p)$ ,  $n \in \mathbb{Z}_+$ ,  $p \in [0, 1]$ .

PMF:  $p_X(x) = \binom{n}{x} p^x(1-p)^{n-x}$ ,  $x \in \{0, \dots, n\}$ .

MGF:  $M_X(s) = (1 - p + p \exp s)^n$ .

Moments:  $\mathbb{E}[X] = np$ ,  $\text{var } X = np(1-p)$ .

- $X \sim \text{Geometric}(p)$ ,  $p \in (0, 1)$ .

PMF:  $p_X(x) = pq^{x-1}$ ,  $x \in \mathbb{Z}_+$ ,  $q = 1 - p$ .

MGF:  $M_X(s) = (p \exp s)/(1 - q \exp s)$ ,  $s < \ln(1/q)$ .

Moments:  $\mathbb{E}[X] = p^{-1}$ ,  $\text{var } X = q/p^2$ .

- $X \sim \text{Poisson}(\lambda)$ ,  $\lambda > 0$ .

PMF:  $p_X(x) = \lambda^x \exp(-\lambda)/x!$ ,  $x \in \mathbb{N}$ .

MGF:  $M_X(s) = \exp(\lambda(\exp s - 1))$ .

Moments:  $\mathbb{E}[X] = \lambda$ ,  $\text{var } X = \lambda$ .

$X, Y$  independent,  $X \sim \text{Poisson}(\lambda)$ ,  $Y \sim \text{Poisson}(\mu) \implies X + Y \sim \text{Poisson}(\lambda + \mu)$ .

- $X \sim \text{Uniform}[a, b]$ ,  $a < b$ .

PDF:  $f_X(x) = (b - a)^{-1}$ ,  $x \in [a, b]$ .

MGF:  $M_X(s) = (\exp(sb) - \exp(sa))/(s(b - a))$ .

Moments:  $\mathbb{E}[X] = (a + b)/2$ ,  $\text{var } X = (b - a)^2/12$ .

- $X \sim \text{Exponential}(\lambda)$ ,  $\lambda > 0$ .

PDF:  $f_X(x) = \lambda \exp(-\lambda x)$ ,  $x > 0$ .

CDF:  $F_X(x) = (1 - \exp(-\lambda x)) \mathbb{1}_{\{x \geq 0\}}$ .

MGF:  $M_X(s) = \lambda/(\lambda - s)$ ,  $s < \lambda$ .

Moments:  $\mathbb{E}[X] = \lambda^{-1}$ ,  $\text{var } X = \lambda^{-2}$ .

- $X \sim \mathcal{N}(\mu, \sigma^2)$ ,  $\mu \in \mathbb{R}$ ,  $\sigma^2 > 0$ .

PDF:  $f_X(x) = (\sqrt{2\pi}\sigma)^{-1} \exp(-(x - \mu)^2/(2\sigma^2))$ .

CDF:  $F_X(x) = \Phi(x)$ .

MGF:  $M_X(s) = \exp(\mu s + \sigma^2 s^2/2)$ .

Moments:  $\mathbb{E}[X] = \mu$ ,  $\text{var } X = \sigma^2$ .

$X, Y$  independent,  $X \sim \mathcal{N}(\mu_1, \sigma_1^2)$ ,  $Y \sim \mathcal{N}(\mu_2, \sigma_2^2) \implies X + Y \sim \mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$ .

## 2 Definitions & Equations

Tail Sum: For  $X \geq 0$ ,  $\mathbb{E}[X] = \int_0^\infty \mathbb{P}(X \geq x) dx$ .

Variance:  $\text{var } X = \mathbb{E}[(X - \mathbb{E}[X])^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$ . Sum:  $\text{var } \sum_{i=1}^n X_i = \sum_{i=1}^n \text{var } X_i + \sum_{i \neq j} \text{cov}(X_i, X_j)$ .

Covariance:  $\text{cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$ . Matrix: If  $X = (X_1, \dots, X_n)$ ,  $(\text{cov } X)_{i,j} = \text{cov}(X_i, X_j)$ .

Correlation:  $\rho(X, Y) = \text{cov}(X, Y)/\sqrt{(\text{var } X)(\text{var } Y)}$ .

Entropy:  $H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_2 p(x) = -\mathbb{E}[\log_2 p(X)]$ .

Order Statistics:  $f_{X^{(i)}}(x) = n \binom{n-1}{i-1} f(x) F(x)^{i-1} (1 - F(x))^{n-i}$ .

MGF:  $M_X(s) = \mathbb{E}[\exp(sX)]$ .

Markov: For  $X \geq 0$ ,  $x > 0$ ,  $\mathbb{P}(X \geq x) \leq \mathbb{E}[X]/x$ .

Chebyshev: For  $x > 0$ ,  $\mathbb{P}(|X - \mathbb{E}[X]| \geq x) \leq (\text{var } X)/x^2$ .