

Electrical Engineering 126: Probability & Random Processes

Midterm 2 Cheat Sheet

Fall 2018

1 Distributions

- $X \sim \text{Bernoulli}(p)$, $p \in [0, 1]$.

PMF: $p_X(x) = p^x(1-p)^{1-x}$, $x \in \{0, 1\}$.

MGF: $M_X(s) = 1 - p + p \exp s$.

Moments: $\mathbb{E}[X] = p$, $\text{var } X = p(1-p)$.

- $X \sim \text{Binomial}(n, p)$, $n \in \mathbb{Z}_+$, $p \in [0, 1]$.

PMF: $p_X(x) = \binom{n}{x} p^x(1-p)^{n-x}$, $x \in \{0, \dots, n\}$.

MGF: $M_X(s) = (1 - p + p \exp s)^n$.

Moments: $\mathbb{E}[X] = np$, $\text{var } X = np(1-p)$.

- $X \sim \text{Geometric}(p)$, $p \in (0, 1)$.

PMF: $p_X(x) = pq^{x-1}$, $x \in \mathbb{Z}_+$, $q = 1 - p$.

MGF: $M_X(s) = (p \exp s)/(1 - q \exp s)$, $s < \ln(1/q)$.

Moments: $\mathbb{E}[X] = p^{-1}$, $\text{var } X = q/p^2$.

- $X \sim \text{Poisson}(\lambda)$, $\lambda > 0$.

PMF: $p_X(x) = \lambda^x \exp(-\lambda)/x!$, $x \in \mathbb{N}$.

MGF: $M_X(s) = \exp(\lambda(\exp s - 1))$.

Moments: $\mathbb{E}[X] = \lambda$, $\text{var } X = \lambda$.

X, Y independent, $X \sim \text{Poisson}(\lambda)$, $Y \sim \text{Poisson}(\mu) \implies X + Y \sim \text{Poisson}(\lambda + \mu)$.

- $X \sim \text{Uniform}[a, b]$, $a < b$.

PDF: $f_X(x) = (b-a)^{-1}$, $x \in [a, b]$.

MGF: $M_X(s) = (\exp(sb) - \exp(sa))/(s(b-a))$.

Moments: $\mathbb{E}[X] = (a+b)/2$, $\text{var } X = (b-a)^2/12$.

- $X \sim \text{Exponential}(\lambda)$, $\lambda > 0$.

PDF: $f_X(x) = \lambda \exp(-\lambda x)$, $x > 0$.

CDF: $F_X(x) = (1 - \exp(-\lambda x))\mathbb{1}_{\{x \geq 0\}}$.

MGF: $M_X(s) = \lambda/(\lambda - s)$, $s < \lambda$.

Moments: $\mathbb{E}[X] = \lambda^{-1}$, $\text{var } X = \lambda^{-2}$.

- $X \sim \mathcal{N}(\mu, \sigma^2)$, $\mu \in \mathbb{R}$, $\sigma^2 > 0$.

PDF: $f_X(x) = (\sqrt{2\pi}\sigma)^{-1} \exp(-(x-\mu)^2/(2\sigma^2))$.

CDF: $F_X(x) = \Phi(x)$.

MGF: $M_X(s) = \exp(\mu s + \sigma^2 s^2/2)$.

Moments: $\mathbb{E}[X] = \mu$, $\text{var } X = \sigma^2$.

X, Y independent, $X \sim \mathcal{N}(\mu_1, \sigma_1^2)$, $Y \sim \mathcal{N}(\mu_2, \sigma_2^2) \implies X + Y \sim \mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$.

Continued:

- $X \sim \text{Pascal}(k, p)$, $k \in \mathbb{Z}_+$, $p \in (0, 1)$.

Sum of k i.i.d. Geometric(p).

PMF: $p_X(x) = \binom{x-1}{k-1} p^k (1-p)^{x-k}$, $x = k, k+1, k+2, \dots$

- $X \sim \text{Erlang}(k, \lambda)$, $k \in \mathbb{Z}_+$, $\lambda > 0$.

Sum of k i.i.d. Exponential(λ).

PDF: $f_X(x) = \lambda^k x^{k-1} \exp(-\lambda x)/(k-1)!$, $x \geq 0$.

2 Definitions & Equations

Tail Sum: For $X \geq 0$, $\mathbb{E}[X] = \int_0^\infty \mathbb{P}(X \geq x) dx$.

Variance: $\text{var } X = \mathbb{E}[(X - \mathbb{E}[X])^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$. Sum: $\text{var } \sum_{i=1}^n X_i = \sum_{i=1}^n \text{var } X_i + \sum_{i \neq j} \text{cov}(X_i, X_j)$.

Covariance: $\text{cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$. Matrix: If $X = (X_1, \dots, X_n)$, $(\text{cov } X)_{i,j} = \text{cov}(X_i, X_j)$.

Correlation: $\rho(X, Y) = \text{cov}(X, Y)/\sqrt{(\text{var } X)(\text{var } Y)}$.

Entropy: $H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_2 p(x) = -\mathbb{E}[\log_2 p(X)]$.

Order Statistics: $f_{X^{(i)}}(x) = n \binom{n-1}{i-1} f(x) F(x)^{i-1} (1 - F(x))^{n-i}$.

MGF: $M_X(s) = \mathbb{E}[\exp(sX)]$.

Markov: For $X \geq 0$, $x > 0$, $\mathbb{P}(X \geq x) \leq \mathbb{E}[X]/x$.

Chebyshev: For $x > 0$, $\mathbb{P}(|X - \mathbb{E}[X]| \geq x) \leq (\text{var } X)/x^2$.

Chernoff: For $t > 0$, $\mathbb{P}(X \geq x) = \mathbb{P}(e^{tX} \geq e^{tx})$.

For $t > 0$, $\mathbb{P}(X \leq x) = \mathbb{P}(e^{-tX} \geq e^{-tx})$.