UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion Session I
Fall 2018

Problem 1. Consider a sphere that has $\frac{1}{10}$ of its surface colored blue, and the rest is colored red. Show that, no matter how the colors are distributed, it is possible to inscribe a cube in the sphere with all of its vertices red.

Problem 2. You know that, at least one of the events A_r (for $r \in \{1, \ldots, n\}$, where n is an integer ≥ 2) is certain to occur but certainly no more than two occur. Show that if the probability of occurrence of any single event is p, and the probability of joint occurrence of any two distinct events is q, we have $p \geq 1/n$ and $q \leq 2/[n(n-1)]$.

Problem 3. Let $n \in \mathbb{Z}_{>1}$. You throw n balls, one after the other, into n bins, so that each ball lands in one of the bins uniformly at random. What is an appropriate sample space to model this scenario? What is the probability that exactly one bin is empty?

Problem 4. (**Bonus**): Prove the **Borel-Cantelli Lemma**: If $A_1, A_2, \ldots$ is a sequence of events with $\sum_{i=1}^{\infty} \Pr(A_i) < \infty$, then

$$\Pr(\text{infinitely many of } A_1, A_2, \ldots \text{ occur}) = 0.$$