

EE126: PROBABILITY AND RANDOM PROCESSES

Discussion Session 10

Fall 2018

Problem 1. Lazy Server

Customers arrive at a service facility at the times of a Poisson process of rate λ . The service facility has infinite capacity. There is an infinitely powerful but lazy server who visits the service facility at the times of a Poisson process of rate μ . The Poisson process of server visits is independent of the Poisson process of arrival times of the customers. When the server visits the facility she instantaneously serves all the customers that are currently waiting in the facility and then immediately leaves (until her next visit).

Thus, for instance, at any time, any customers that are waiting in the service facility would only be those that arrived after the most recent visit of the server.

1. Show that the system admits a well-defined stationary regime for all values of the parameters $\lambda > 0$ and $\mu > 0$.
2. Find the mean number of customers waiting in the system at any given time in the stationary regime.

Problem 2. Frogs

Three frogs are playing near a pond. When they are in the sun they get too hot and jump in the lake at rate 1. When they are in the lake they get too cold and jump onto the land at rate 2. The rates here refer to the rate in exponential distribution. Let X_t be the number of frogs in the sun at time $t \geq 0$.

- (a) Find the stationary distribution for $(X_t)_{t \geq 0}$.
- (b) Check the answer to (a) by noting that the three frogs are independent two-state Markov chains.

Problem 3. Poisson Queues

A continuous-time queue has Poisson arrivals with rate λ , and it is equipped with infinitely many servers. The servers can work in parallel on multiple customers, but they are non-cooperative in the sense that a single customer can only be served by one server. Thus, when there are k customers in the queue ($k \in \mathbb{N}$), k servers are active. Suppose that the service time of each customer is exponentially distributed with rate μ and they are i.i.d.

- (a) Argue that the queue-length is a Markov chain. Draw the transition diagram of the Markov chain.

- (b) Prove that for all finite values of λ and μ the Markov chain is positive-recurrent and find the invariant distribution.

Problem 4. Machine

A machine, once in production mode, operates continuously until an alarm signal is generated. The time up to the alarm signal is an exponential random variable with parameter 1. Subsequent to the alarm signal, the machine is tested for an exponentially distributed amount of time with parameter 5. The test results are positive, with probability $1/2$, in which case the machine returns to production mode, or negative, with probability $1/2$, in which case the machine is taken for repair. The duration of the repair is exponentially distributed with parameter 3.

- (a) Let states 1, 2, 3 correspond to production mode, testing, and repair, respectively. Let $(X(t))_{t \geq 0}$ denote the state of the system at time t . Is $(X(t))_{t \geq 0}$ a CTMC?
- (b) Find the rate and transition matrices.
- (c) Find the steady state probabilities.