UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: PROBABILITY AND RANDOM PROCESSES

Discussion Section 11
Fall 2018

Problem 1. **Statistical Estimation**
Given $X \in \{0,1\}$, the random variable Y is exponentially distributed with rate $3X+1$.

(a) Assume $\Pr(X = 1) = p \in (0,1)$ and $\Pr(X = 0) = 1 - p$. Find the MAP estimate of X given Y.

(b) Find the MLE of X given Y.

Problem 2. **Exponential: MLE & MAP**
The random variable X is exponentially distributed with mean 1. Given X, the random variable Y is exponentially distributed with rate X.

1. Find $MLE[X \mid Y]$.
2. Find $MAP[X \mid Y]$.

Problem 3. **Laplace Prior & ℓ^1-Regularization**
Suppose you draw n i.i.d. data points $(x_1, y_1), \dots, (x_n, y_n)$, where n is a positive integer and the true relationship is $Y = WX + \varepsilon$, $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. (That is, Y has a linear dependence on X, with additive Gaussian noise.) Further suppose that W has a prior distribution with density

$$f_W(w) = \frac{1}{2\beta} \exp^{-|w|/\beta}, \qquad \beta > 0.$$

(This is known as the **Laplace distribution**.) Show that finding the MAP estimate of W given the data points $\{(x_i, y_i) : i = 1, \dots, n\}$ is equivalent to minimizing the cost function

$$J(w) = \sum_{i=1}^{n} (y_i - wx_i)^2 + \lambda |w|$$

(you should determine what λ is). This is interpreted as a one-dimensional ℓ^1-regularized least-squares criterion, also known as LASSO.