UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion Session 12
Fall 2018

Problem 1. **Hypothesis Testing for Gaussian Distribution**
Assume that X has prior probabilities $\Pr(X = 0) = \Pr(X = 1) = 1/2$. Further

- If $X = 0$, then $Y \sim \mathcal{N}(\mu_0, \sigma_0^2)$.
- If $X = 1$, then $Y \sim \mathcal{N}(\mu_1, \sigma_1^2)$.

Assume $\mu_0 < \mu_1$ and $\sigma_0 < \sigma_1$.
Using the Bayesian formulation of hypothesis testing, find the optimal *decision rule* $r : \mathbb{R} \to \{0, 1\}$ with respect to the minimum expected cost criterion

$$\min_{r:\mathbb{R}\to\{0,1\}} \mathbb{E}[I\{r(Y) \neq X\}].$$

Problem 2. **Hypothesis Testing for Uniform Distribution**
Assume that

- If $X = 0$, then $Y \sim \text{Uniform}[-1, 1]$.
- If $X = 1$, then $Y \sim \text{Uniform}[0, 2]$.

Using the Neyman-Pearson formulation of hypothesis testing, find the optimal randomized *decision rule* $r : [-1, 2] \to \{0, 1\}$ with respect to the criterion

$$\begin{aligned} \min_{\text{randomized } r:[-1,2]\to\{0,1\}} \quad & \Pr\big(r(Y) = 0 \bigm| X = 1\big) \\ \text{s.t.} \quad & \Pr\big(r(Y) = 1 \bigm| X = 0\big) \le \beta, \end{aligned}$$

where $\beta \in [0, 1/2]$ is a given upper bound on the false positive probability.

Problem 3. **Gaussian LLSE**
The random variables X, Y, Z are i.i.d. $\mathcal{N}(0, 1)$.

(a) Find $L[X^2 + Y^2 \mid X + Y]$.

(b) Find $L[X + 2Y \mid X + 3Y + 4Z]$.

(c) Find $L[(X + Y)^2 \mid X - Y]$.